1. **Question Details**

Prove the identity.

\[ \tanh(ln \ x) = \frac{x^2 - 1}{x^2 + 1} \]

\[ \tanh(ln \ x) = \frac{\cosh(ln \ x)}{x - (e^{ln \ x})^{-1}} = \frac{\frac{e^{ln \ x} - (e^{ln \ x})^{-1}}{2} \cdot \frac{e^{ln \ x} + (e^{ln \ x})^{-1}}{2}}{\frac{x - (e^{ln \ x})^{-1}}{x + x^{-1}}} = \frac{x - 1/x}{x + 1/x} = \frac{(x^2 - 1)/x}{(x^2 + 1)/x} = \frac{x^2 - 1}{x^2 + 1} \]
Use the definitions of the hyperbolic functions to find each of the following limits.

(a) \( \lim_{x \to \infty} \tanh x \)

(b) \( \lim_{x \to -\infty} \tanh x \)

(c) \( \lim_{x \to \infty} \sinh x \)

(d) \( \lim_{x \to -\infty} \sinh x \)

(e) \( \lim_{x \to \infty} \sech x \)

(f) \( \lim_{x \to \infty} \coth x \)

(g) \( \lim_{x \to 0^+} \coth x \)

(h) \( \lim_{x \to 0^-} \coth x \)

(i) \( \lim_{x \to -\infty} \csch x \)
Prove this equation using the method of this example and a previous equation with \( x \) replaced by \( y \).

(a) using the method of the example

Let \( y = \tanh^{-1} x \). Then

\[
= \tanh y = \frac{\sinh y}{\cosh y} = \frac{e^y - 1}{(e^y + e^{-y})/2} \Rightarrow \frac{e^y}{e^y + 1} \Rightarrow xe^{2y} + x = e^{2y} - 1
\]

\[
\Rightarrow 1 + x = \frac{e^{2y} - 1}{e^{2y} + 1} - xe^{2y}
\]

\[
\Rightarrow 1 + x = e^{2y}(1 - x)
\]

\[
\Rightarrow e^{2y} = \frac{1 + x}{1 + x}
\]

\[
2y = \ln\left(\frac{1 + x}{1 - x}\right)
\]

\[
\Rightarrow y = \ldots
\]

(b) using the method of a previous equation

Let \( y = \tanh^{-1} x \). Then \( x \), so we have \( e^{2y} = \frac{1 + \tanh y}{1 - \tanh y} = \frac{1 + x}{1 - x} \)

\[
2y = \ln\left(\frac{1 + x}{1 - x}\right)
\]

\[
\Rightarrow y = \ldots
\]

Solution or Explanation

(a) Let \( y = \tanh^{-1} x \). Then

\[
x = \tanh y = \frac{\sinh y}{\cosh y} = \frac{(e^y - e^{-y})/2}{(e^y + e^{-y})/2} \Rightarrow \frac{e^y}{e^y + 1} \Rightarrow xe^{2y} + x = e^{2y} - 1 \Rightarrow 1 + x = e^{2y} - xe^{2y} \Rightarrow 1 + x = e^{2y}(1 - x)
\]

\[
e^{2y}(1 - x) \Rightarrow e^{2y} = \frac{1 + x}{1 - x} \Rightarrow 2y = \ln\left(\frac{1 + x}{1 - x}\right) \Rightarrow y = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right).
\]

(b) Let \( y = \tanh^{-1} x \). Then \( x = \tanh y \), so from Exercise 18 we have

\[
e^{2y} = \frac{1 + \tanh y}{1 - \tanh y} = \frac{1 + x}{1 - x} \Rightarrow 2y = \ln\left(\frac{1 + x}{1 - x}\right) \Rightarrow y = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right).
\]

4. Find the derivative. Simplify where possible.

\[
g(x) = \cosh(\ln x)
\]

\[
g'(x) = \ldots
\]

Solution or Explanation

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5. **Question Details**

Find the derivative. Simplify where possible.

\[ y = x \coth(6 + x^2) \]

\[ y'(x) = \]

**Solution or Explanation**

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6. **Question Details**

Find the differential of each function.

(a) \[ y = x^2 \sin 4x \]

\[ dy = \]

(b) \[ y = \sqrt{5 + t^2} \]

\[ dy = \]

**Solution or Explanation**

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7. **Question Details**

Find the linearization \( L(x) \) of the function at \( a \).

\[ f(x) = x^{1/2}, \ a = 25 \]

\[ L(x) = \]

**Solution or Explanation**

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8. **Question Details**

Compute \( \Delta y \) and \( dy \) for the given values of \( x \) and \( dx = \Delta x \). (Round your answers to three decimal places.)

\[ y = \frac{6}{x}, \ x = 4, \ \Delta x = 1 \]

\[ \Delta y = \]

\[ dy = \]

Sketch a diagram showing the line segments with lengths \( dx, dy, \) and \( \Delta y \).
Solution or Explanation

\[ y = f(x) = \frac{6}{x}, \ x = 4, \ \Delta x = 1 \Rightarrow \]

\[ \Delta y = f(5) - f(4) = \frac{6}{5} - \frac{6}{4} = -0.3 \]

\[ dy = -\frac{6}{x^2}, \ \Delta x = -\frac{6}{4^2}(1) = -0.375 \]
9. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemispherical dome with diameter 58 m. (Round your answer to two decimal places.)

\[ m^3 \]

Solution or Explanation

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10. Find the linearization \( L(x) \) of the function at \( a \).

\[ f(x) = \cos x, \ a = \frac{5\pi}{2} \]

\[ L(x) = \]

Solution or Explanation

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11. A trough is 12 ft long and its ends have the shape of isosceles triangles that are 4 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 9 ft\(^3\)/min, how fast is the water level rising when the water is 9 inches deep?

\[ \text{ft/min} \]

Solution or Explanation

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Two carts, A and B, are connected by a rope 39 ft long that passes over a pulley P (see the figure). The point Q is on the floor $h = 12$ ft directly beneath $P$ and between the carts. Cart A is being pulled away from Q at a speed of 2 ft/s. How fast is cart B moving toward Q at the instant when cart A is 5 ft from Q? (Round your answer to two decimal places.)

ft/s

Solution or Explanation
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