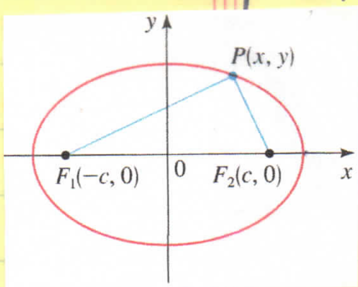
§6.2

Ellipses: An ellipse is the set of all points in the plane the sum of whose distances from two fixed points F_1 and F_2 is constant. $F_1 \wedge F_2$ are the foci.



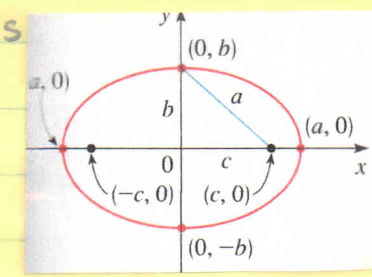
$$d(P,F_1)+d(P,F_2)=2a$$
$$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a$$
$$\sqrt{(x-c)^2+y^2}=2a-\sqrt{(x+c)^2+y^2}$$
$$x^2-2cx+c^2+y^2=4a^2-4a\sqrt{(x+c)^2+y^2}+(x^2+2cx+c^2+y^2)$$
$$4a\sqrt{(x+c)^2+y^2}=4a^2+4cx$$
$$a^2\left[(x+c)^2+y^2\right]=(a^2+cx)^2$$
$$a^2x^2+2a^2cx+a^2c^2+a^2y^2=a^4+2a^2cx+c^2x^2$$
$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$

The sum of the distances from P to the foci must be larger than the distance between the foci $\Rightarrow 2a>2c$, $a>c$

$\therefore a^2-c^2>0$. Divide by $a^2(a^2-c^2)$:

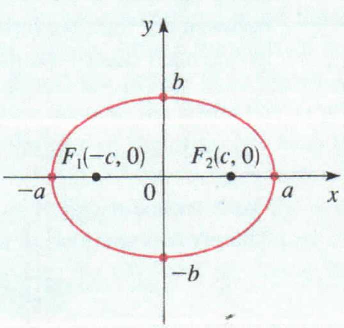
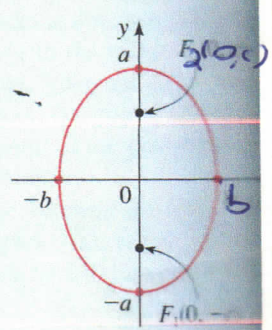
$\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1$. Let $b^2=a^2-c^2 \wedge b^2<a^2 \Rightarrow b<a$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$; major vs minor axes



ELLIPSE WITH CENTER AT THE ORIGIN

The graph of each of the following equations is an ellipse with center at the origin and having the given properties.

EQUATION	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
	$a>b>0$	$a>b>0$
VERTICES	$(\pm a, 0)$	$(0, \pm a)$
MAJOR AXIS	Horizontal, length $2a$	Vertical, length $2a$
MINOR AXIS	Vertical, length $2b$	Horizontal, length $2b$
FOCI	$(\pm c, 0)$, $c^2=a^2-b^2$	$(0, \pm c)$, $c^2=a^2-b^2$
GRAPH		

$\frac{x^2}{9}+\frac{y^2}{4}=1$ Foci $(\pm 5, 0)$

vertices $(\pm 3, 0)$

major axis: 6, minor: 4

The vertices of an ellipse are $(\pm 4, 0) \wedge$ foci are $(\pm 2, 0)$. Find eqn.

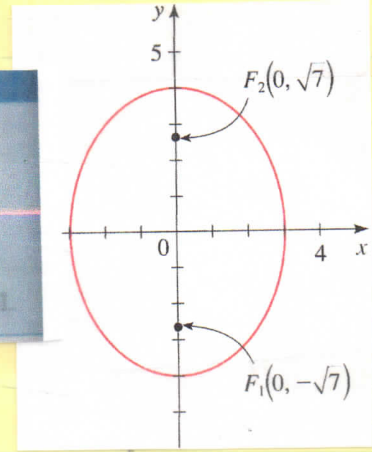
$a=4$, $c=2$, $b^2=12$

$\frac{x^2}{16}+\frac{y^2}{12}=1$

Find the foci of ellipse: $16x^2 + 9y^2 = 144$

$\frac{x^2}{9} + \frac{y^2}{16} = 1$, $16 > 9 \Rightarrow$ foci on y-axis w/ $a=4$, $b=3$

$c^2 = a^2 - b^2 = 16 - 9 = 7$; $c = \sqrt{7}$, $F(0, \pm\sqrt{7})$



Eccentricities of the Orbits of the Planets

The orbits of the planets are ellipses with the sun at one focus. For most planets these ellipses have very small eccentricity, so they are nearly circular. However, Mercury and Pluto, the innermost and outermost known planets, have visibly elliptical orbits.

Planet	Eccentricity
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.056
Uranus	0.046
Neptune	0.010
Pluto	0.248

DEFINITION OF ECCENTRICITY

For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ (with $a > b > 0$), the **eccentricity** e is the number

$$e = \frac{c}{a}$$

where $c = \sqrt{a^2 - b^2}$. The eccentricity of every ellipse satisfies $0 < e < 1$.

Find the equation of the ellipse w/ foci $(0, \pm 8)$ ∧ eccentricity $e = 4/5$

ans: $c = 8$, $e = \frac{c}{8}$ $\frac{4}{5} = \frac{8}{a}$ $a = 10$ $b^2 = a^2 - c^2 \Rightarrow b = 6$

$\frac{x^2}{36} + \frac{y^2}{100} = 1$

{evens 2-50}