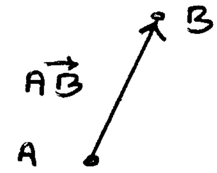
3.5 5 Vectors

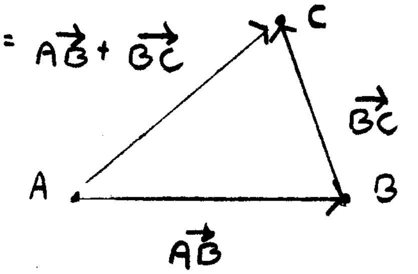
$\vec{u} = \vec{AB}$



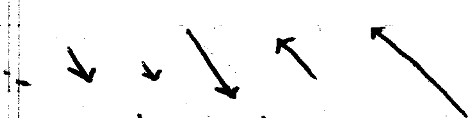

A vector in the plane is a line segment with an assigned direction. Point A is the initial point and B is the terminal point.

$|\vec{AB}|$ is the magnitude

$\vec{AC} = \vec{AB} + \vec{BC}$




Multiplication of a vector by a scalar

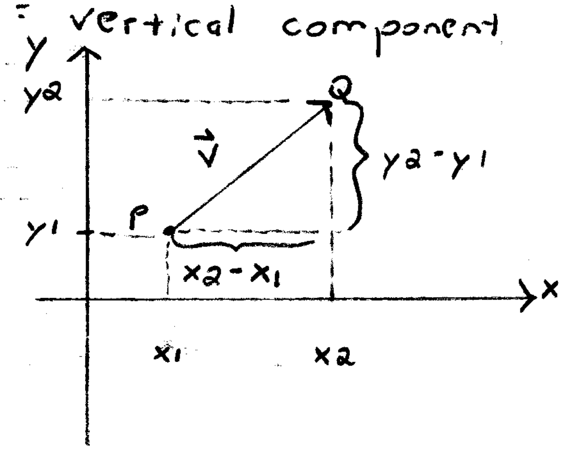


$\vec{v}$ $\frac{1}{2}\vec{v}$ $2\vec{v}$ $-\vec{v}$ $-4/3\,\vec{v}$

$v = \langle a, b \rangle$

$a$ = horizontal component

$b$ = vertical component



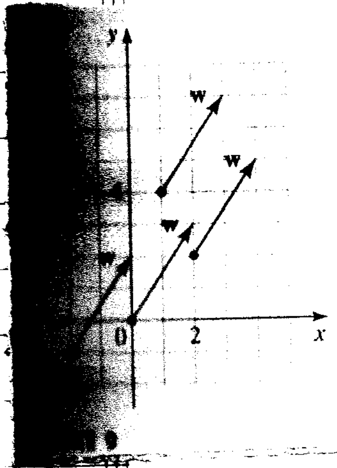

## COMPONENT FORM OF A VECTOR

If a vector **v** is represented in the plane with initial point $P(x_1, y_1)$ and terminal point $Q(x_2, y_2)$, then

$$\mathbf{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

**EXAMPLE 1** ■ **Describing Vectors in Component Form**

(a) Find the component form of the vector **u** with initial point $(-2, 5)$ and terminal point $(3, 7)$.
(b) If the vector $\mathbf{v} = \langle 3, 7 \rangle$ is sketched with initial point $(2, 4)$, what is its terminal point?
(c) Sketch representations of the vector $\mathbf{w} = \langle 2, 3 \rangle$ with initial points at $(0, 0)$, $(2, 2)$, $(-2, -1)$, and $(1, 4)$.




SOLUTION

(a) The desired vector is

$$\mathbf{u} = \langle 3 - (-2), 7 - 5 \rangle = \langle 5, 2 \rangle$$

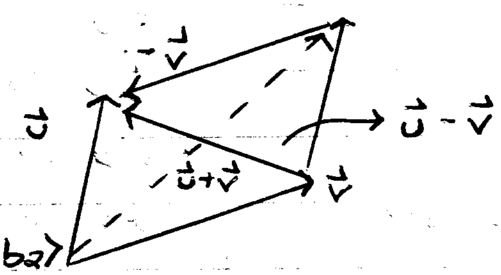
(b) Let the terminal point of **v** be $(x, y)$. Then

$$\langle x - 2, y - 4 \rangle = \langle 3, 7 \rangle$$

So $x - 2 = 3$ and $y - 4 = 7$, or $x = 5$ and $y = 11$. The terminal point is $(5, 11)$.

(c) Representations of the vector **w** are sketched in Figure 9. ■

Subtraction of Vectors




If $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$

$\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$

$\vec{u} - \vec{v} = \langle a_1 - a_2, b_1 - b_2 \rangle$

$c\vec{u} = \langle ca_1, cb_1 \rangle, c \in R$

$|\vec{v}| = \sqrt{a_2^2 + b_2^2}$

Unit vector: a vector of length 1

$\vec{w} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

**PROPERTIES OF VECTORS**

| Vector addition | Multiplication by a scalar |
|---|---|
| $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ | $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$ |
| $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ | $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$ |
| $\mathbf{u} + \mathbf{0} = \mathbf{u}$ | $(cd)\mathbf{u} = c(d\mathbf{u}) = d(c\mathbf{u})$ |
| $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$ | $1\mathbf{u} = \mathbf{u}$ |
| **Length of a vector** | $0\mathbf{u} = \mathbf{0}$ |
| $\lvert c\mathbf{u} \rvert = \lvert c \rvert \, \lvert \mathbf{u} \rvert$ | $c\mathbf{0} = \mathbf{0}$ |

$\vec{i} = \langle 1, 0 \rangle$

$\vec{j} = \langle 0, 1 \rangle$

**VECTORS IN TERMS OF i AND j**

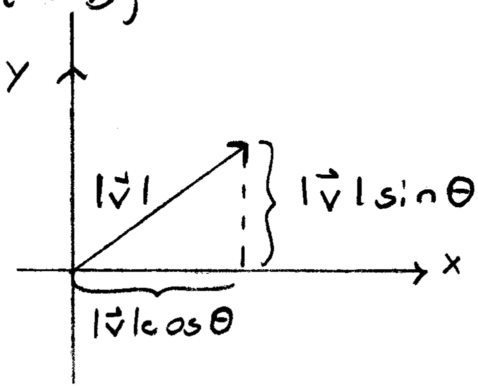
The vector $\mathbf{v} = \langle a, b \rangle$ can be expressed in terms of **i** and **j** by

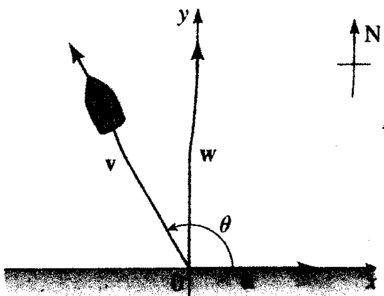
$$\mathbf{v} = \langle a, b \rangle = a\mathbf{i} + b\mathbf{j}$$

Horizontal & Vertical Components of a Vector

Let $\vec{v}$ be a vector with magnitude $|v|$ and direction $\theta$. Then $\vec{v} = \langle a, b \rangle = a\hat{\imath} + b\hat{\jmath}$ :

$a = |v| \cos\theta$ , $b = |v| \sin\theta$




A woman launches a boat from one shore of a straight river and wants to land at the point directly on the opposite shore. If the speed of the boat relative to the water is 10mph and the river is flowing east at 5 mph, in what direction should she head the boat in order to arrive at the desired landing point?

1. Set up coordinate system as shown in picture
2. $\vec{u}$ = velocity of river $\vec{v}$ = velocity of boat , $|v| = 10$
3. $\vec{u} = 5\hat{\imath}$ $\quad \vec{v} = (10\cos\theta)\hat{\imath} + (10\sin\theta)\hat{\jmath}$
4. $\vec{w} = \vec{u} + \vec{v} = 5\hat{\imath} + 10\cos\theta\,\hat{\imath} + 10\sin\theta\,\hat{\jmath}$

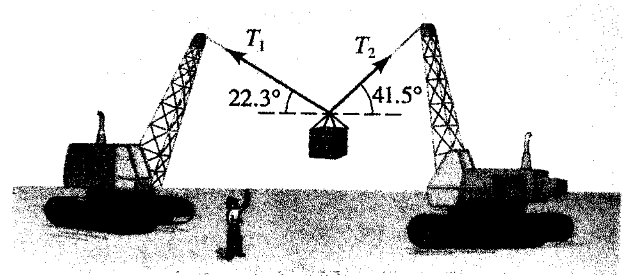
$= (5 + 10\cos\theta)\hat{\imath} + 10\sin\theta\,\hat{\jmath}$

$5 + 10\cos\theta = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^\circ$

**58.** The cranes in the figure are lifting an object that weighs 18,278 lb. Find the tensions $\mathbf{T}_1$ and $\mathbf{T}_2$.



From the figure we see that $\mathbf{T}_1 = -|\mathbf{T}_1|\cos 22.3°\,\mathbf{i} + |\mathbf{T}_1|\sin 22.3°\,\mathbf{j}$ and
$\mathbf{T}_2 = |\mathbf{T}_2|\cos 41.5°\,\mathbf{i} + |\mathbf{T}_2|\sin 41.5°\,\mathbf{j}$. Since $\mathbf{T}_1 + \mathbf{T}_2 = 18{,}278\mathbf{j}$ we get
$-|\mathbf{T}_1|\cos 22.3° + |\mathbf{T}_2|\cos 41.5° = 0$ and $|\mathbf{T}_1|\sin 22.3° + |\mathbf{T}_2|\sin 41.5° = 18{,}278$.
From the first equation $|\mathbf{T}_2| = |\mathbf{T}_1|\,\dfrac{\cos 22.3°}{\cos 41.5°}$ and substituting into the second equation gives
$|\mathbf{T}_1|\sin 22.3° + |\mathbf{T}_1|\,\dfrac{\cos 22.3°\sin 41.5°}{\cos 41.5°} = 18{,}278 \quad\Leftrightarrow$
$|\mathbf{T}_1|(\sin 22.3°\cos 41.5° + \cos 22.3°\sin 41.5°) = 18{,}278\cos 41.5° \quad\Leftrightarrow$
$|\mathbf{T}_1|\sin(22.3° + 41.5°) = 18{,}278\cos 41.5° \quad\Leftrightarrow\quad |\mathbf{T}_1| = 18{,}278\,\dfrac{\cos 41.5°}{\sin 63.8°} \approx 15{,}257$. Similarly,
solving for $|\mathbf{T}_1|$ in the first equation gives $|\mathbf{T}_1| = |\mathbf{T}_2|\,\dfrac{\cos 41.5°}{\cos 22.3°}$ and substituting gives
$|\mathbf{T}_2|\,\dfrac{\cos 41.5°\sin 22.3°}{\cos 22.3°} + |\mathbf{T}_2|\sin 41.5° = 18{,}278 \quad\Leftrightarrow$
$|\mathbf{T}_2|(\sin 22.3°\cos 41.5° + \sin 41.5°\cos 22.3°) = 18{,}278\cos 22.3° \quad\Leftrightarrow$
$|\mathbf{T}_2| = \dfrac{18{,}278\cos 22.3°}{\sin 63.8°} \approx 18{,}847$.
Thus $\mathbf{T}_1 \approx (-15{,}257\cos 22.3°)\mathbf{i} + (15{,}257\sin 22.3°)\mathbf{j} \approx -14{,}116\mathbf{i} + 5{,}789\mathbf{j}$ and
$\mathbf{T}_2 \approx (18{,}847\cos 41.5°)\mathbf{i} + (18{,}847\sin 41.5°)\mathbf{j} \approx 14{,}116\mathbf{i} + 12{,}488\mathbf{i}$.