§5.4

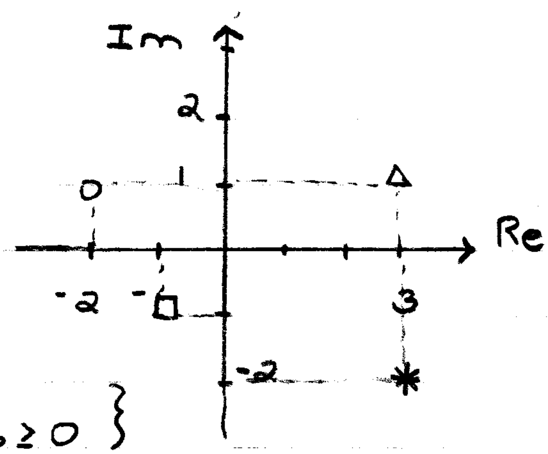
De Moivre's Theorem

Graph $z_1 = 3 - 2i$ *

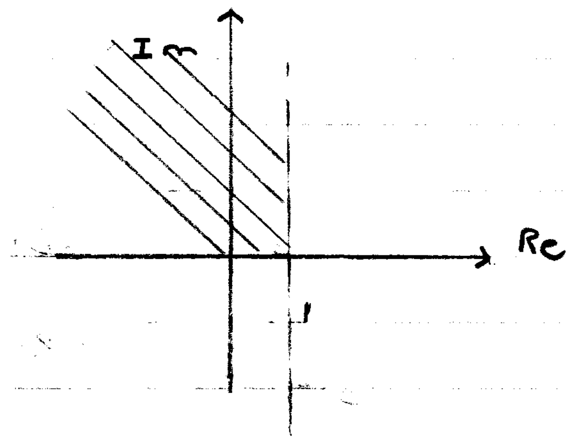
$z_2 = 3 + i$ △

$z_3 = -i - 1$ □

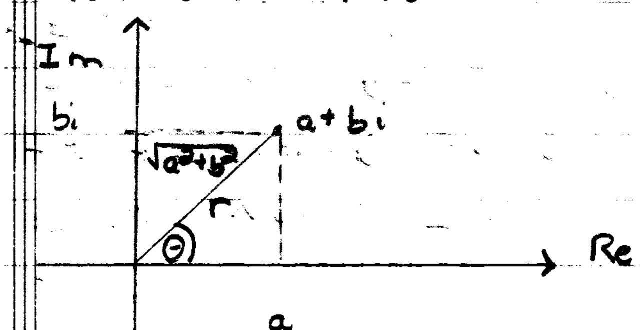
$z_4 = -2 + i$ ○



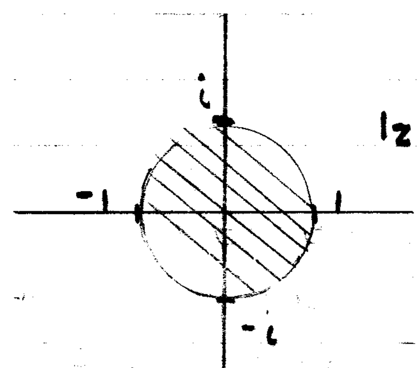
Graph $T = \{a + bi : a < 1, b \geq 0\}$



Absolute Value



$|z| = \sqrt{a^2 + b^2}$

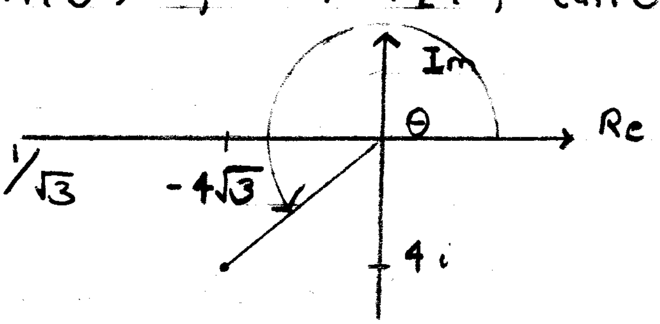


$|z| \overset{r}{=} 1$

Polar Form of Complex Numbers

$z = a + bi$

$= r(\cos\theta + i\sin\theta)$; $r = |z|$, $\tan\theta = \frac{b}{a}$



$-4\sqrt{3} - 4i$

$\tan\theta = {}^{-4}/_{-4\sqrt{3}} = {}^{1}/_{\sqrt{3}}$

$\theta = {}^{7\pi}/_{6}$

$r = 8$

Thus $-4\sqrt{3} - 4i = 8\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

$-1 + \sqrt{3}i$:

$\tan\theta = \sqrt{3}/-1 = -\sqrt{3} \Rightarrow \theta = \frac{2\pi}{3}$

$r = \sqrt{1+3} = 2$

Thus $-1 + \sqrt{3}i = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ $\quad z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$

$z_1/z_2 = r_1/r_2\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$

proof p. 323

De Moivre's Thm

If $z = r(\cos\theta + i\sin\theta)$, $\forall n \in \mathbb{Z}$,

$z^n = r^n(\cos n\theta + i\sin n\theta)$

Proof: p. 324

$\left(\frac{1}{2} + \frac{1}{2}i\right)^{10} = \left[\frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{10}$

$= \left(\frac{\sqrt{2}}{2}\right)^{10}\left(\cos\frac{10\pi}{4} + i\sin\frac{5\pi}{2}\right) = \frac{1}{32}i$

n^{th} Roots of Complex Numbers

$Z = r(\cos\theta + i\sin\theta)$

$w_k = r^{1/n}\left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right]$

$k = 0, 1, 2, \ldots, n-1$ proof p 325

Find six sixth roots of -64

$Z = 64(\cos\pi + i\sin\pi)$

$w_k = 64^{1/6}\left[\cos\left(\frac{\pi + 2k\pi}{6}\right) + i\sin\left(\frac{\pi + 2k\pi}{6}\right)\right]$

$k = 0, 1, 2, 3, 4, 5$

$w_0 = 64^{1/6}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{3} + i$

$w_1 = 64^{1/6}\,\text{cis}\,\frac{\pi}{2} = 2i$

$w_2 = 64^{1/6}\,\text{cis}\,\frac{5\pi}{6} = -\sqrt{3} + i$

$w_3 = 64^{1/6}\,\text{cis}\,\frac{7\pi}{6} = -\sqrt{3} - i$

$w_4 = 64^{1/6}\,\text{cis}\,\frac{3\pi}{2} = -2i$

$w_5 = 64^{1/6}\,\text{cis}\,\frac{11\pi}{6} = \sqrt{3} - i$