§5.3
§5.4

Complex Numbers

A complex number is an expression of the form

$a + bi$; $a, b \in \mathbb{R}$; $i^2 = -1$

The real part is a.

The imaginary part is b

$i = \sqrt{-1}$

Equality of two complex numbers

$a + bi = c + di$ if (1) $a = c$ and
(2) $b = d$

1.
$$\begin{array}{r} a + bi \\ + \; c + di \\ \hline (a+c) + (b+d)i \end{array}$$

2.
$$\begin{array}{r} a + bi \\ - \; c + di \\ \hline (a-c) + (b-d)i \end{array}$$

3.
$$\begin{array}{rr} a & + \; bi \\ c & + \; di \\ \hline adi & - bd \\ ac + cbi & \\ \hline \end{array}$$
$(ac - bd) + i(ad + bc)$

Complex Conjugate

$z = a + bi$

$\bar{z} = a - bi$

$z\bar{z} = a^2 + b^2$

4. $\dfrac{a+bi}{c+di} = \dfrac{a+bi}{c+di} \cdot \dfrac{c-di}{c-di} = \dfrac{(ac+bd)+(bc-ad)i}{c^2+d^2}$

5. Powers of i

$i^{23} = i^{20+3} = (i^2)^{10} i^3 = (-1)^{10} i^3 = i^3 = -i$

Complex Roots of Quadratic Equations

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

if $z = \alpha + \beta i$ is a solution to $ax^2 + bx + c = 0$, then so is $\bar{z} = \alpha - \beta i$

Examples (work in class)

1. $(-2 + i)(3 - 7i)$
2. $(2 - 3i)^{-1}$
3. i^{1002}
4. $\frac{\sqrt{-7}\sqrt{-49}}{\sqrt{28}}$
5. $x^2 + \frac{1}{2}x + 1$
6. $2x + 3 = 2x$
7. $\overline{zw} = \bar{z}\,\bar{w}$