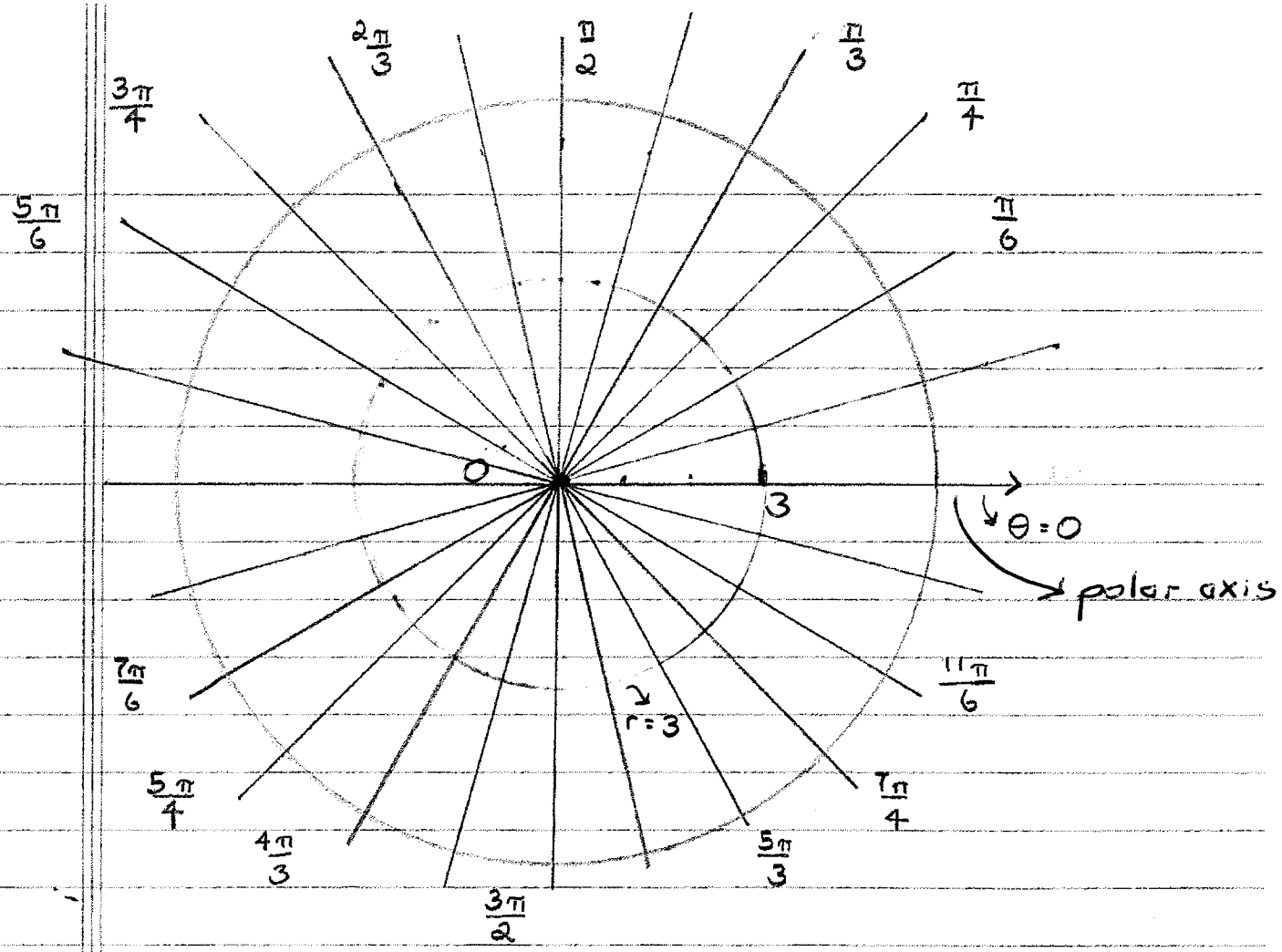


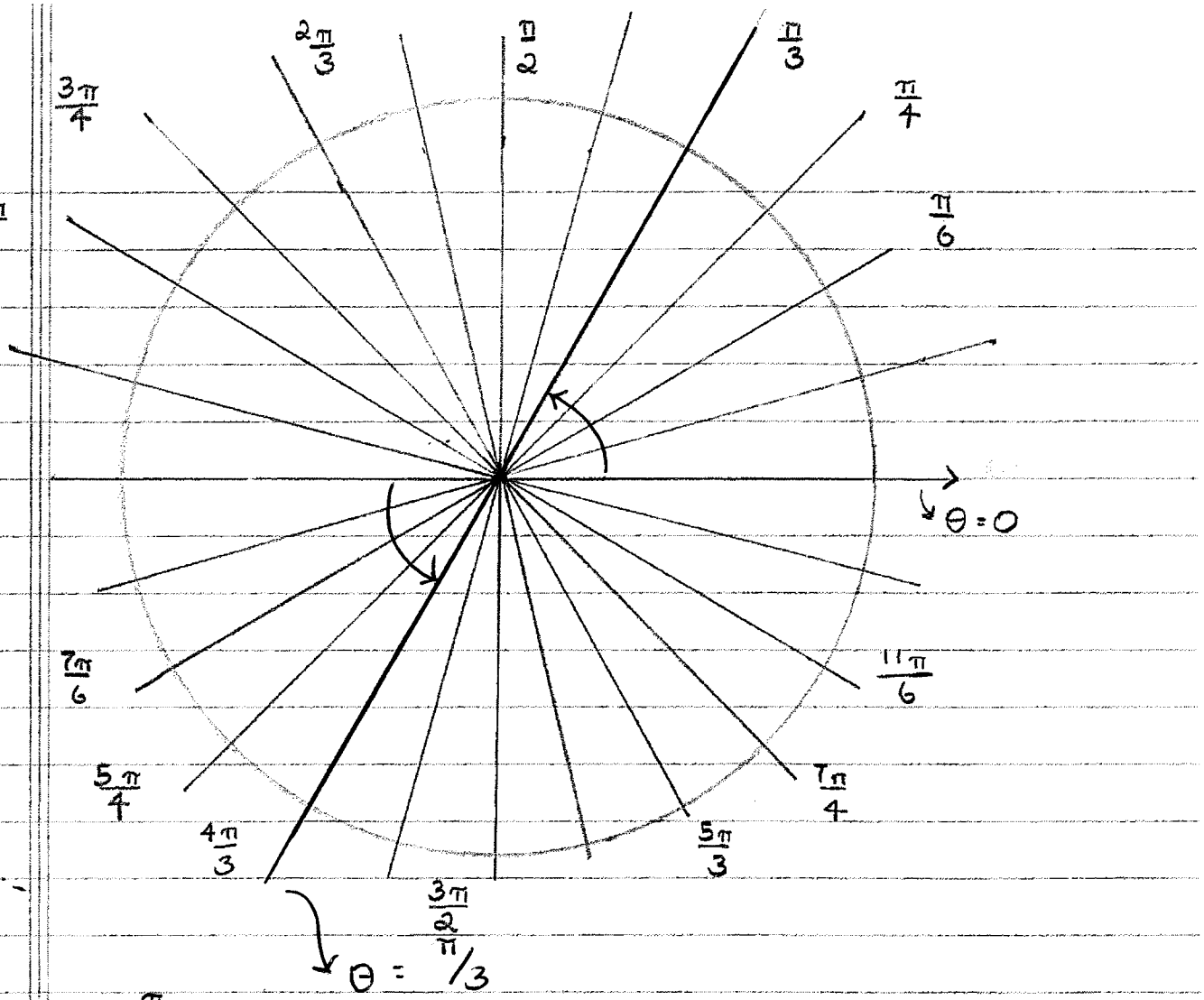
$r = 3$

Symmetry Tests

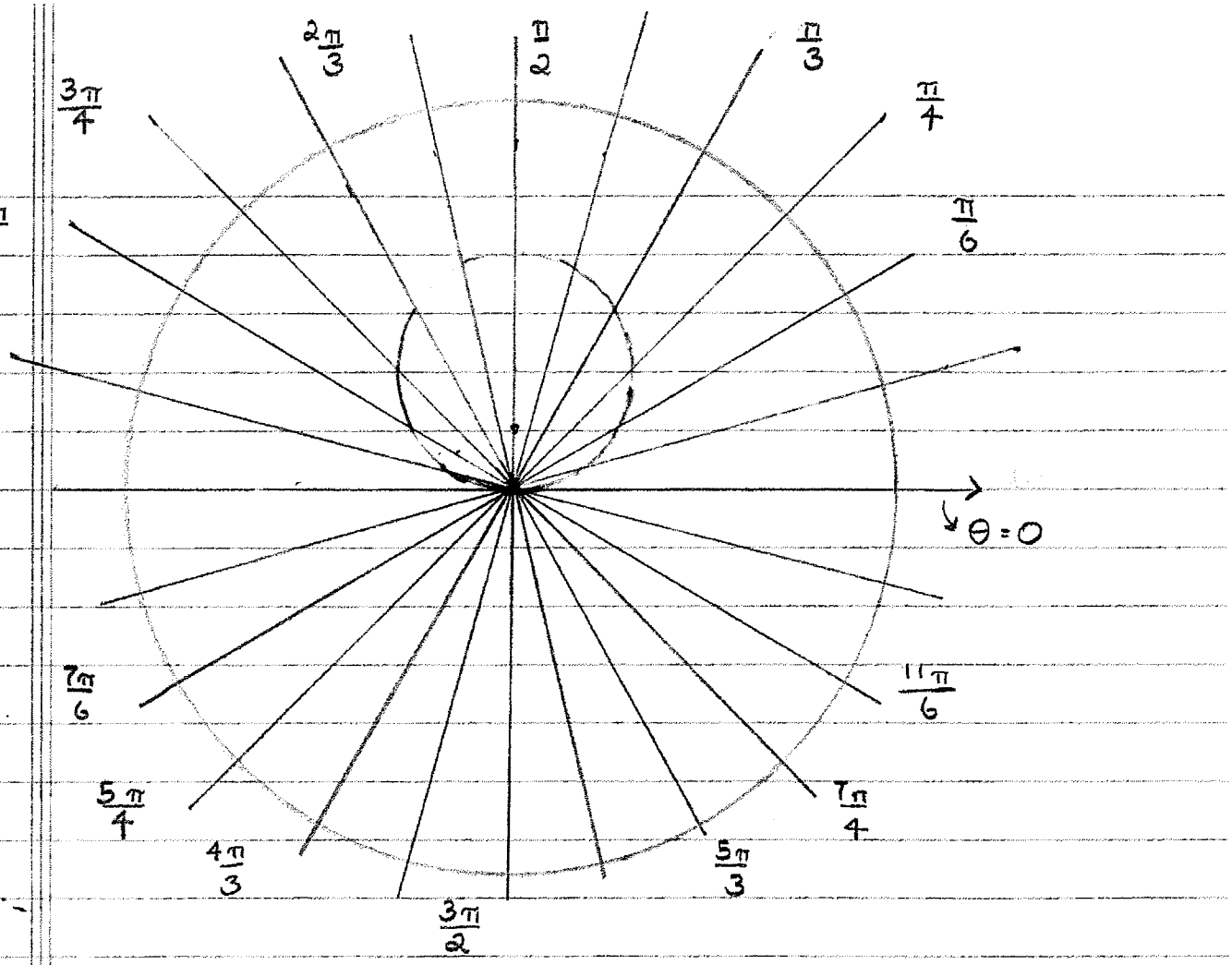
1 If a polar equation is unchanged when we replace θ by $-\theta$, then the graph is symmetric about the polar axis

2 If the equation is unchanged when we replace r by $-r$, then the graph is symmetric about the pole

3 If the graph is ~~symmetric~~ unchanged when we replace θ by $\pi - \theta$, the graph is symmetric about the vertical line $\theta = \pi/2$ (the y-axis



$\theta = \frac{\pi}{3}$



$r = 2\sin\theta$

$r^2 = 2r\sin\theta$

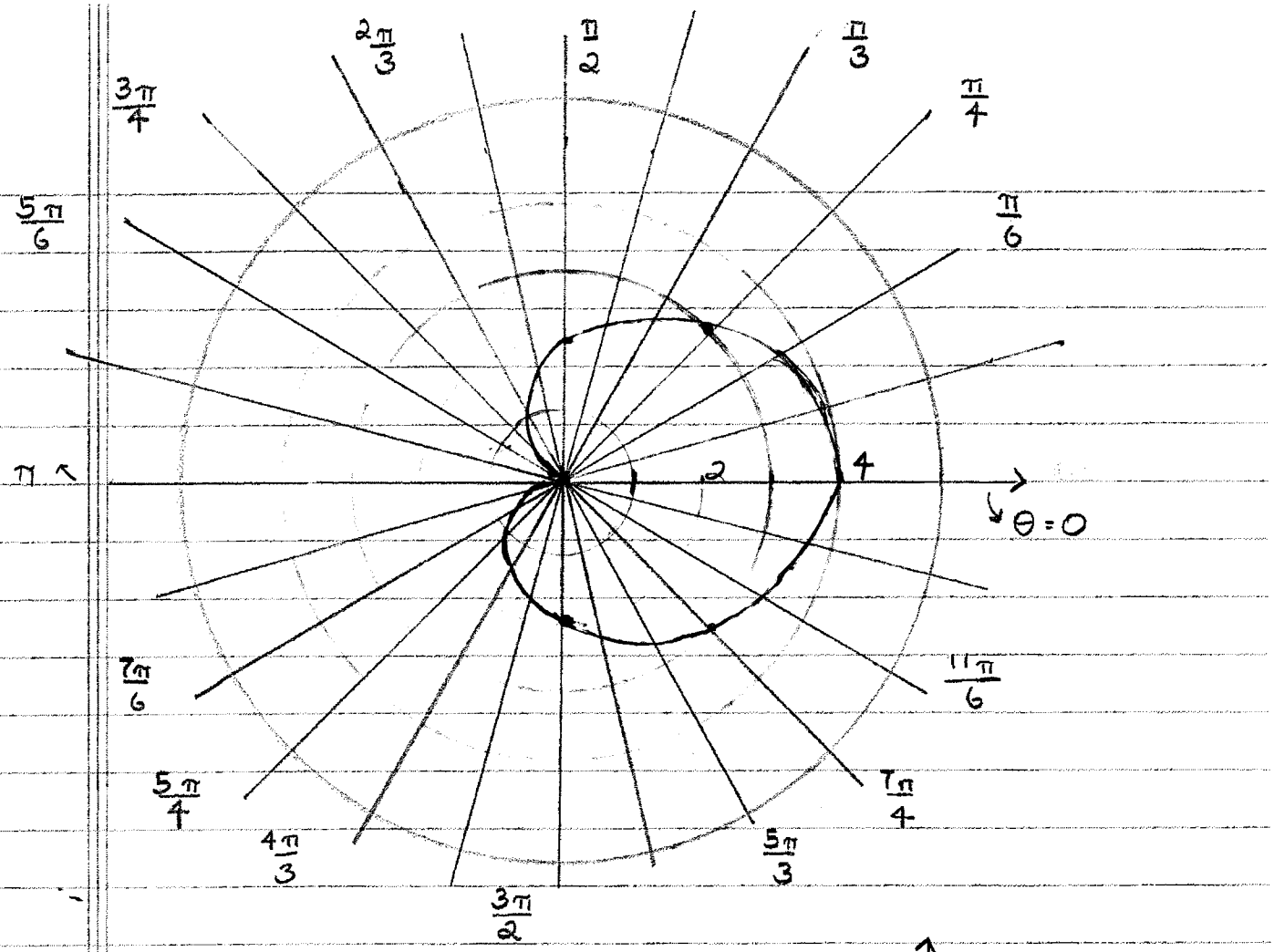
$x^2 + y^2 = 2y$

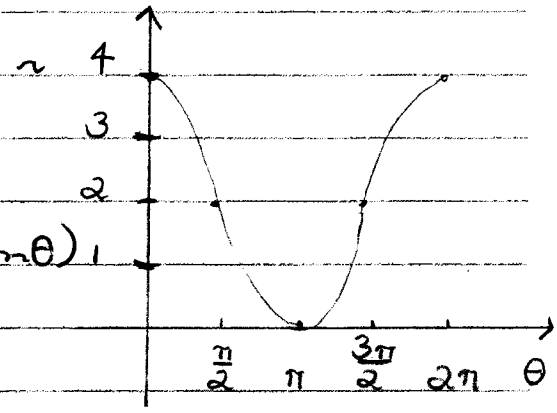
$x^2 + y^2 - 2y = 0$

$x^2 + (y-1)^2 = 1$

$r = 2a\sin\theta$: circle radius $|a|$ centered at $(a, \frac{\pi}{2})$

$r = 2a\cos\theta$: circle radius $|a|$ centered at $(a, 0)$





$r = 2 + 2\cos\theta$

This is a cardiod

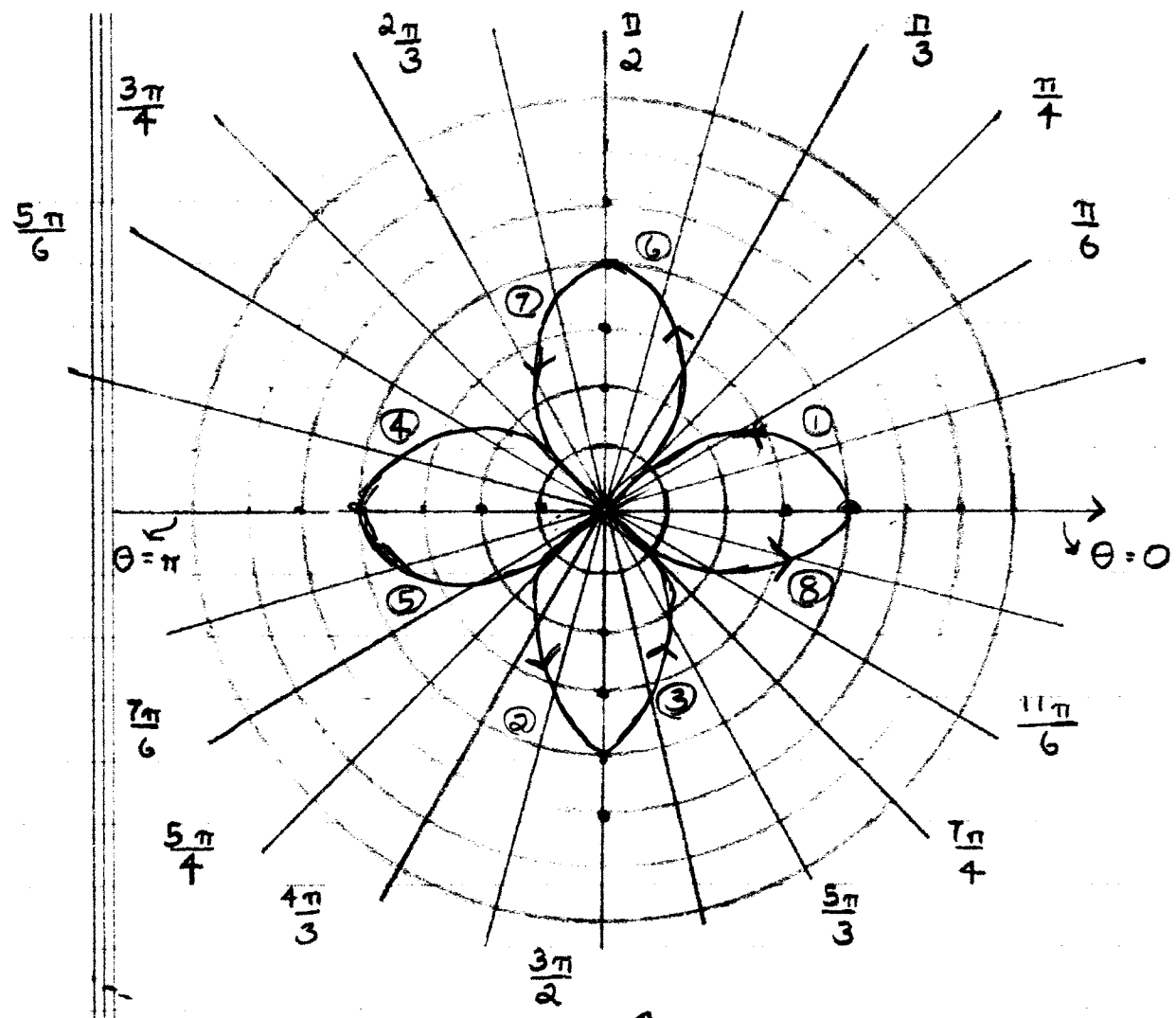
$r = a(1 \pm \cos\theta)$ or $r = a(1 \pm \sin\theta)$

$r^2 = 2r + 2r\cos\theta$

$x^2 + y^2 = 2r + 2x$

$x^2 + y^2 - 2x = 2r$

$(x^2 + y^2 - 2x)^2 = 4r^2 = 4(x^2 + y^2)$



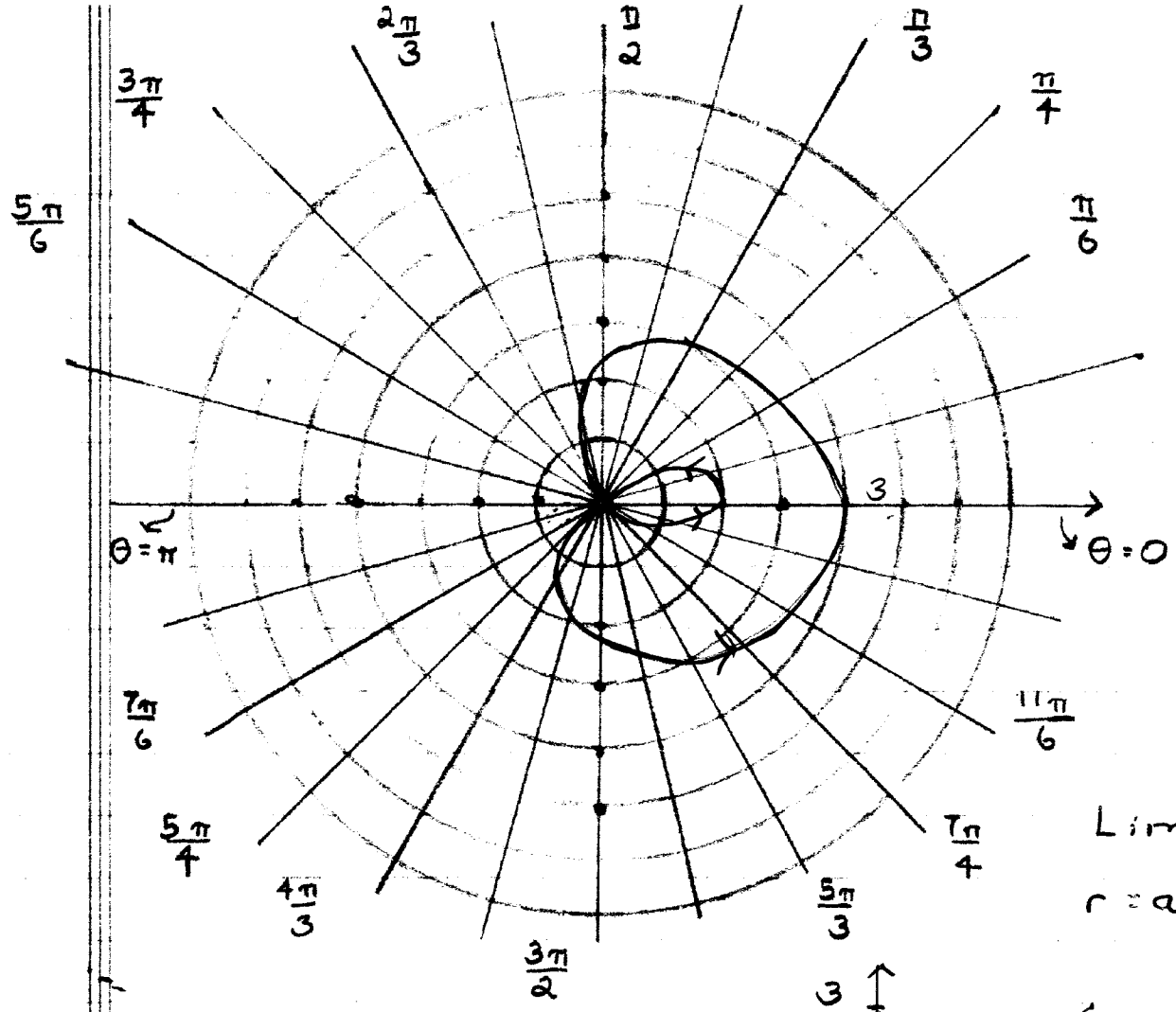
$r = \cos 2\theta$

$r = a\cos n\theta$

$r = a\sin n\theta$

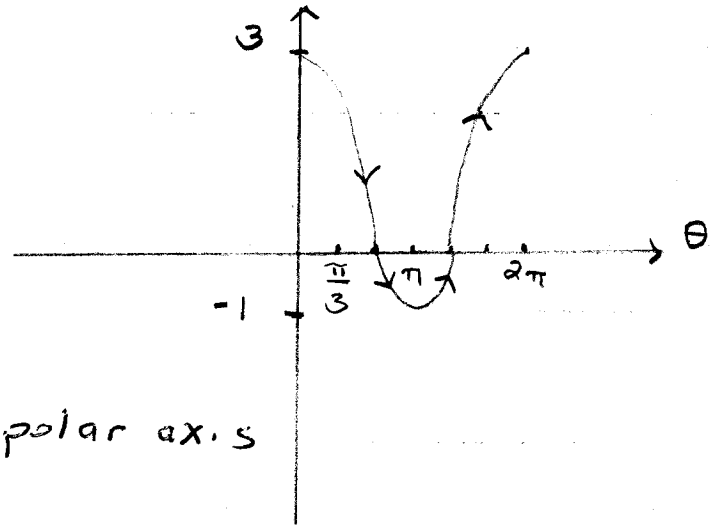
n - odd : n-leafed rose

n - even : $2n$-leafed rose



Limaçon

$r = a \pm b\cos\theta$



$r = 1 + 2\cos\theta$

Symmetry

$r(\theta) = r(-\theta)$

symmetric about polar axis

zeros:

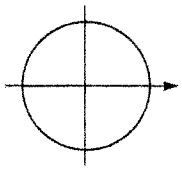
$0 = 1 + 2\cos\theta$

$\cos\theta = -1/2$

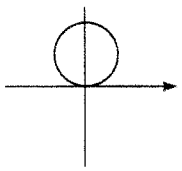
$\theta = 2\pi/3, \frac{4\pi}{3}$

SOME COMMON POLAR CURVES

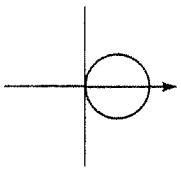
Circles and Spiral



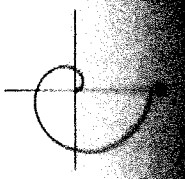
$r = a$
circle



$r = a \sin\theta$
circle



$r = a \cos\theta$
circle



$r = a\theta$
spiral

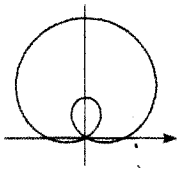
Limaçons

$r = a \pm b \sin\theta$

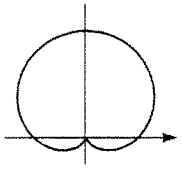
$r = a \pm b \cos\theta$

$(a > 0, b > 0)$

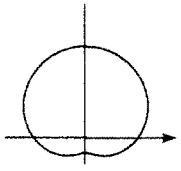
Orientation depends on the trigonometric function (sine or cosine) and the sign of b.



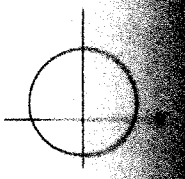
$a < b$
limaçon with inner loop



$a = b$
cardioid



$a > b$
dimpled limaçon



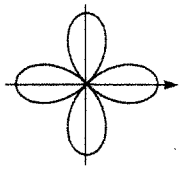
$a > 2b$
convex limaçon

Roses

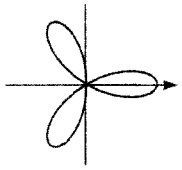
$r = a \sin n\theta$

$r = a \cos n\theta$

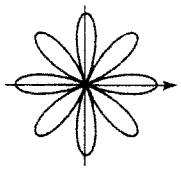
n-leaved if n is odd
$2n$-leaved if n is even



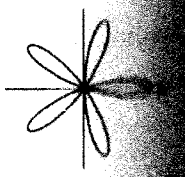
$r = a \cos 2\theta$
4-leaved rose



$r = a \cos 3\theta$
3-leaved rose



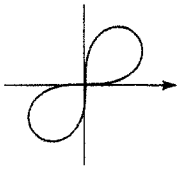
$r = a \cos 4\theta$
8-leaved rose



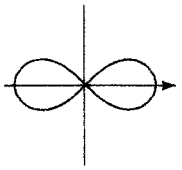
$r = a \cos 5\theta$
5-leaved rose

Lemniscates

Figure-eight-shaped curves



$r^2 = a^2 \sin 2\theta$
lemniscate



$r^2 = a^2 \cos 2\theta$
lemniscate