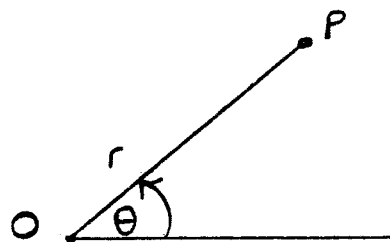


# Chapter 5: Polar Coordinates, Complex Numbers, Vectors

§ 5.1

## Polar Coordinates

What is  $r$ ?  $\theta$ ?

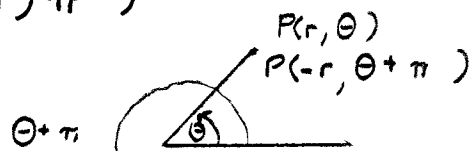


$$P(r, \theta + 2n\pi) = P(-r, \theta + (2n+1)\pi)$$

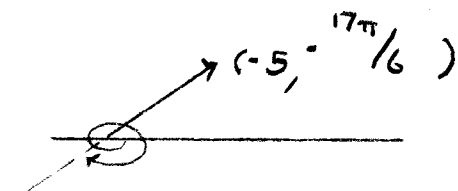
$$\curvearrowright \theta > 0 \quad \curvearrowleft \theta < 0$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$r^2 = x^2 + y^2 \quad \tan \theta = \frac{y}{x}$$



Plot  $(-5, -\frac{17\pi}{6})$

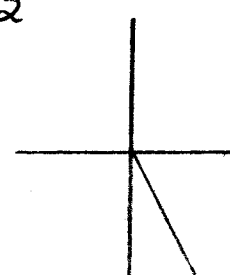


$$(r, \theta) = (\sqrt{3}, -\frac{5\pi}{3}) \Rightarrow x = r \cos \theta = \sqrt{3} \cos(-\frac{5\pi}{3}) = \frac{3}{2}$$

$$y = r \sin \theta = \sqrt{3} \sin(-\frac{5\pi}{3}) = -\frac{3\sqrt{3}}{2}$$

$$(x, y) = (1, -2) \Rightarrow r = \sqrt{1+4} = \sqrt{5} \quad \theta = \tan^{-1} \frac{-2}{1}$$

$$0 \leq \theta < 2\pi \Rightarrow \theta = 2\pi + \tan^{-1}(-2)$$



$$(x, y) = (3\sqrt{3}, -3) \Rightarrow r = \sqrt{27+9} = 6 \quad \theta = \tan^{-1}(-\frac{1}{\sqrt{3}}) = \theta = \frac{11\pi}{6}$$

$$y = 5 \quad \theta = ? \quad r = ? \quad y = r \sin \theta \quad x = 5, \quad \theta = \frac{\pi}{2}$$

$$x^2 + y^2 = 9 \quad r \sin \theta = 5, \quad r = \frac{5}{\sin \theta}$$

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 9 \quad r = 3$$

$$\sqrt{x^2 - y^2} = 1 \quad r^2 \cos^2 \theta - r^2 \sin^2 \theta = 1$$

$$r^2 \cos 2\theta = 1$$

$$r^2 = \sec 2\theta$$

$$\theta = \pi \Leftrightarrow \tan \theta = 0; \frac{y}{x} = 0 \quad y = 0$$

$$r = \frac{1}{1 + \sin \theta} \quad r(1 + \sin \theta) = 1$$

$$r^2 = (1 - r \sin \theta)^2 \quad \Leftrightarrow \quad r + r \sin \theta = 1 \Rightarrow r^2 = 1 - r \sin \theta$$
$$\Leftrightarrow \quad x^2 + y^2 = (1 - y)^2 = 1 - 2y + y^2$$
$$x^2 + 2y - 1 = 0$$

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$$r = 2 - \cos \theta$$

$$r^2 = 2r - r \cos \theta$$

$$r^2 + r \cos \theta = 2r$$

$$(r^2 + r \cos \theta)^2 = 4r^2$$

$$(x^2 + y^2 + x)^2 = 4(x^2 + y^2)$$

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