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Trig Equ

①

$$\csc^2 x - 4 = 0$$

$$\csc x = \pm 2 \Rightarrow \csc x = \pm 2$$

$$\sin x = \pm \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, x \in [0, 2\pi)$$

$$\therefore x = \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi, k \in \mathbb{J}$$

②

$$(2\cos x + \sqrt{3})(2\sin x - 1) = 0$$

$$2\cos x + \sqrt{3} = 0 \quad \text{or} \quad 2\sin x - 1 = 0$$

$$\cos x = -\frac{\sqrt{3}}{2} \quad \text{or} \quad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6} \text{ in } [0, 2\pi)$$

$$x = \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + 2k\pi \quad \forall k \in \mathbb{J}$$

③

$$3\tan^3 x = \tan x \quad 3\tan^3 x - \tan x = 0$$

$$\tan x (3\tan^2 x - 1) = 0$$

$$\tan x = 0 \quad \text{or} \quad 3\tan^2 x - 1 = 0 \Rightarrow \tan x = \pm \frac{1}{\sqrt{3}}$$

$$\downarrow x = k\pi \quad \downarrow x = \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi$$

④

$$\cos x \cos 2x + \sin x \sin 2x = \frac{1}{2}$$

$$\cos(x-2x) = \frac{1}{2} \Leftrightarrow \cos(-x) = \frac{1}{2} \Leftrightarrow \cos(x) = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, x \in [0, 2\pi)$$

⑤

$$\tan x + \cot x = 4 \sin 2x \Leftrightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = 8 \sin x \cos x$$

$$\left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right] \sin x \cos x = (8 \sin x \cos x)(\sin x \cos x)$$

$$\sin x + \cos x = 8 \sin x \cos x$$

$$1 = 2(2 \sin x \cos x) \Leftrightarrow (\sin 2x) = \frac{1}{2}$$

$$\sin 2x = \pm \frac{1}{2}; \quad 2x = \frac{\pi}{4} + k\pi \quad \text{or} \quad 2x = \frac{3\pi}{4} + k\pi$$

$$x = \frac{\pi}{8} + \frac{k\pi}{2} \quad \text{or} \quad x = \frac{3\pi}{8} + \frac{k\pi}{2}$$

$$\text{solutions on } [0, 2\pi): x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$$

$$x = \frac{(2k+1)\pi}{8}, k \in \mathbb{J}$$

$$\begin{aligned} \textcircled{6} \quad \sin 5x - \sin 3x &= \cos 4x \\ 2\cos 4x \sin x &= \cos 4x \\ \cos 4x (2\sin x - 1) &= 0 \\ 4x &= \frac{\pi}{2} + k\pi \quad \text{or} \quad x = \frac{\pi}{6} + 2k\pi, \\ &\quad \frac{5\pi}{6} + 2k\pi \end{aligned}$$

$$\begin{aligned} u + v &= 5x \\ u - v &= 3x \\ 2u &= 8x \quad u = 4x \\ v &= x \end{aligned}$$

$$\therefore x = \frac{\pi}{8} + \frac{k\pi}{4}, \quad \frac{\pi}{6} + 2k\pi, \quad \frac{5\pi}{6} + 2k\pi$$

$$\frac{1}{2} [\sin(u+v) - \sin(u-v)] = \cos u \sin v$$

$$\begin{aligned} \textcircled{7} \quad \sin(\cos x) &= 0 \\ \cos x &= 0 \quad \text{or} \quad \cos x = \pi \quad \text{on} \quad [0, 2\pi) \\ \downarrow x &= \frac{\pi}{2} + k\pi \quad \downarrow \text{no soln} \end{aligned}$$

ch. 4 HW

$$\S 4.1 \quad \{6, 8, 10, 12, 20, 22, 28, 34, 50, 62, 66, 70, 72, 90\}$$

$$\S 4.2 \quad \{2-10 \text{ evens}, 14, 20, 22, 24, 28, 32, 40, 44\}$$

$$\S 4.3 \quad \{6, 8, 12, 18, 28, 34, 40, 46, 50, 58, 64, 70\}$$

$$\S 4.4 \quad \{2-8 \text{ evens}, 14-28 \text{ evens}, 30-40 \text{ evens}, 42-48 \text{ evens}, 52\}$$

$$\S 4.5 \quad \{2-46 \text{ evens}, 56, 58, 60, 64-74 \text{ evens}\}$$