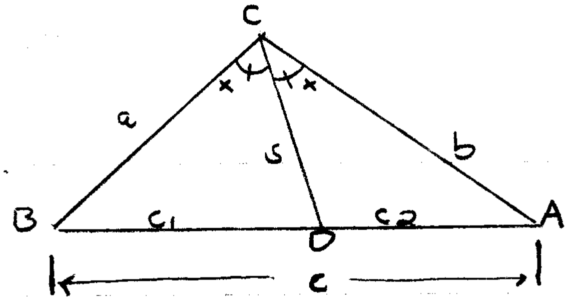
In $\triangle ABC$, line segment s bisects $\angle C$. Find length of s.



Use Law of Sines

$$\frac{c}{\sin 2x} = \frac{b}{\sin B} \quad ; \quad c = \frac{b \sin 2x}{\sin B}$$

For $\triangle BCD$: $\dfrac{s}{\sin B} = \dfrac{c_1}{\sin x}$ $\qquad c_1 = \dfrac{s \sin x}{\sin B}$

For $\triangle ACD$: $\dfrac{s}{\sin A} = \dfrac{c_2}{\sin x}$ $\qquad c_2 = \dfrac{s \sin x}{\sin A}$

$c = c_1 + c_2$: $\quad \dfrac{b \sin 2x}{\sin B} = \dfrac{s \sin x}{\sin B} + \dfrac{s \sin x}{\sin A} = s \sin x \left[\dfrac{1}{\sin B} + \dfrac{1}{\sin A}\right]$

For $\triangle ABC$: $\dfrac{b}{\sin B} = \dfrac{a}{\sin A} \Leftrightarrow \dfrac{1}{\sin A} = \dfrac{b}{a \sin B}$

$$\frac{b \sin 2x}{\sin B} = s \sin x \left[\frac{1}{\sin B} + \frac{b}{a \sin B}\right] = s \sin x \cdot \frac{1}{\sin B} \left[1 + \frac{b}{a}\right]$$

$$b \sin 2x = s(\sin x)\left[\frac{b+a}{a}\right] \Leftrightarrow 2b \sin x \cos x = s \sin x \left[\frac{b+a}{a}\right]$$

$$2b \cos x = s\left[\frac{b+a}{a}\right]$$

$$\frac{2ab \cos x}{b+a} = s$$

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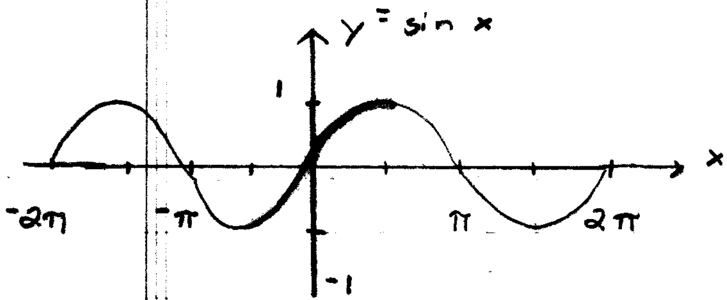
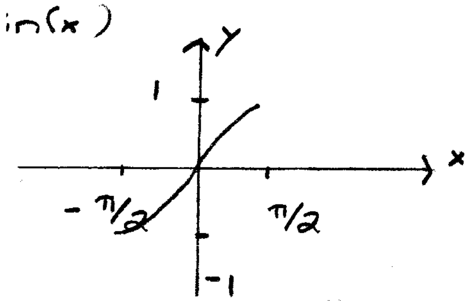
§ 4.4 Inverse Trig Functions

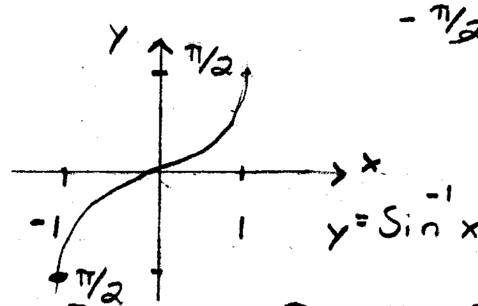
$f^{-1}(x) = y \Leftrightarrow f(y) = x$

Inverse functions must be 1-1. Trig functions are not. Construct "principal parts" of trig functions. These parts are 1-1 ^ are written as Sin(x), Cos(x), Tan(x), Csc(x), Cot(x), Sec(x)

Ranges ~~Domains~~ of each function are same as ranges ~~domains~~ of their counterparts. Their domains are intervals that map x-values in a 1-1 manner to their ranges





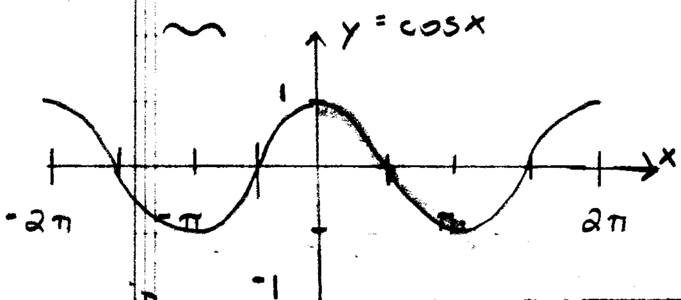


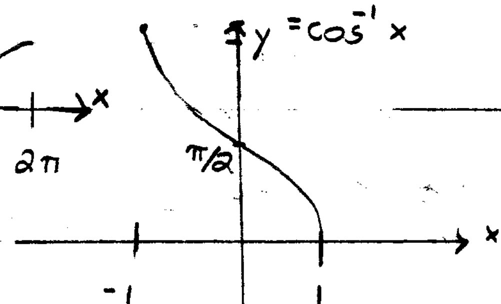
arcsin(x)

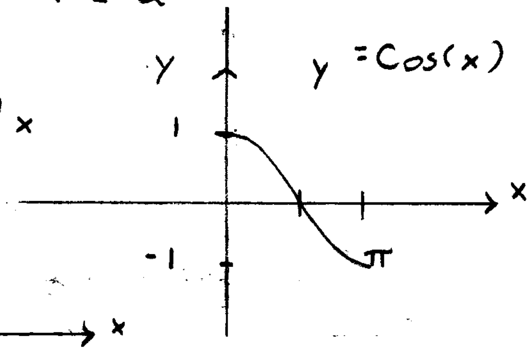
$\sin^{-1} x$: D $[-1, 1]$ R $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin(\sin^{-1} x) = x$ for $-1 \le x \le 1$

$\sin^{-1}(\sin x) = x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$



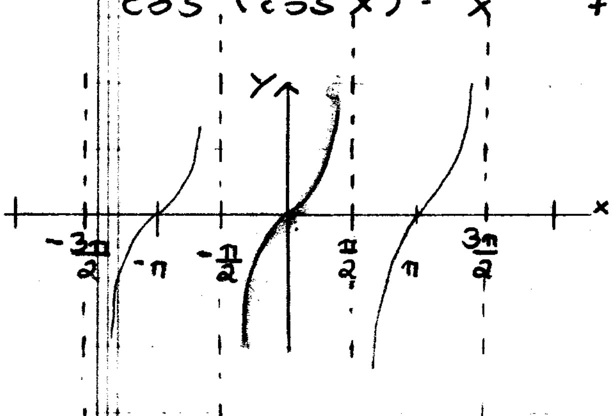


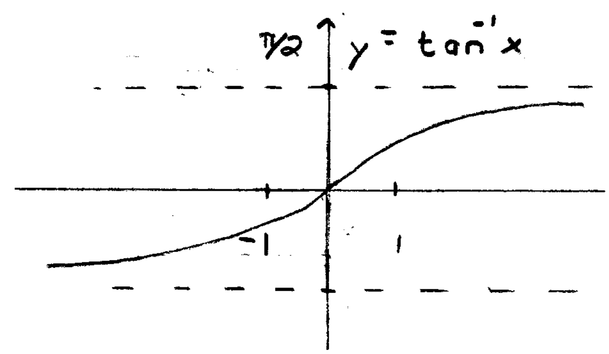


arccos(x) $\cos^{-1} x$: D $[-1, 1]$ R $[0, \pi]$

$\cos(\cos^{-1} x) = x$ for $-1 \le x \le 1$

$\cos^{-1}(\cos x) = x$ for $0 \le x \le \pi$





arctan(x) $\tan^{-1} x$: D : $\mathbb{R}$ R $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\tan(\tan^{-1} x) = x$, $x \in \mathbb{R}$

$\tan^{-1}(\tan x) = x$, $-\pi/2 < x < \pi/2$

* Read text p 268 for discussion regarding remaining inverse trig functions

$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$ $\quad \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$ $\quad \cos^{-1}(-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}$

$\sin^{-1}\sqrt{3}$: not defined $\quad \tan^{-1}0 = 0$

$\tan^{-1}(-1) = -\frac{\pi}{4}$ $\quad \sin(\sin^{-1}5)$: DNE

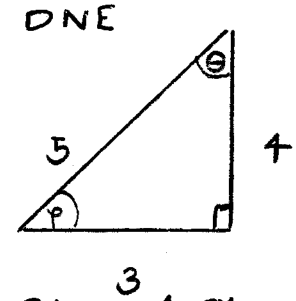
$\sin^{-1}(\sin\frac{5\pi}{6}) = \frac{\pi}{6}$

$\tan^{-1}(-\sqrt{3}) = -\pi/3$

$\cos(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{3}{5}) =$

$\cos(\sin^{-1}\frac{3}{5})\cos(\cos^{-1}\frac{3}{5}) +$

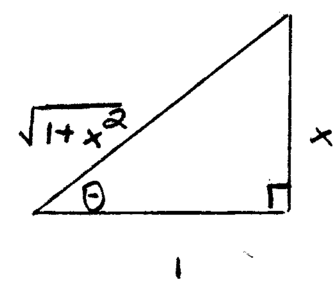
$\quad \sin(\cos^{-1}\frac{3}{5})\sin(\sin^{-1}\frac{3}{5}) = \frac{4}{5}\,\frac{3}{5} + \frac{4}{5}\,\frac{3}{5} = \frac{24}{25}$



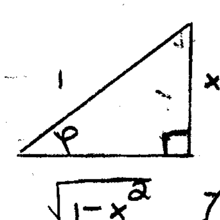
$\sin(\tan^{-1}x - \sin^{-1}x)$

$\sin(\tan^{-1}x)\cos(\sin^{-1}x) -$

$\quad \cos(\tan^{-1}x)\sin(\sin^{-1}x) =$



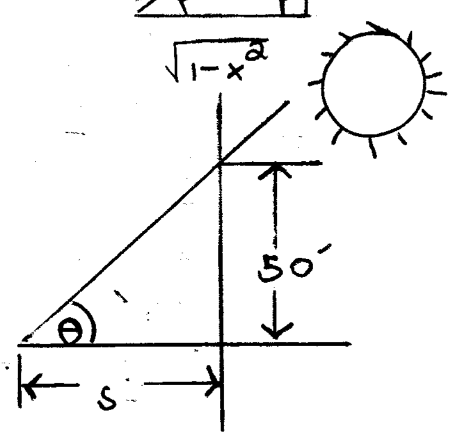
$(\frac{1}{\sqrt{1+x^2}})x\,\sqrt{1-x^2} - \frac{1}{\sqrt{1+x^2}}\,x$



A 50′ pole casts a shadow

Angle of elevation θ of the sun as a function of shadow length s : $\tan\theta = \frac{50}{s}$

When s = 20, $\theta = \tan^{-1}\frac{5}{2} = 1.19$ rad $= ?°$



graph: $y = \tan^{-1} x + \tan^{-1} \frac{1}{x}$

Let $\tan^{-1} x = u \iff \tan u = x \,, \quad -\frac{\pi}{2} < u < \frac{\pi}{2}$

$\frac{1}{x} = \frac{1}{\tan u} = \cot u = \tan\left(\frac{\pi}{2} - u\right)$

$\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = u + \tan^{-1}\left(\tan\left(\frac{\pi}{2} - u\right)\right)$

If $x > 0$, then $0 < u < \frac{\pi}{2}$, so $0 > -u > -\pi/2$

$\frac{\pi}{2} > \frac{\pi}{2} - u > 0$

$\frac{\pi}{2} - u \in QI \Rightarrow \tan^{-1}\left(\tan\left(\frac{\pi}{2} - u\right)\right) = \frac{\pi}{2} - u$

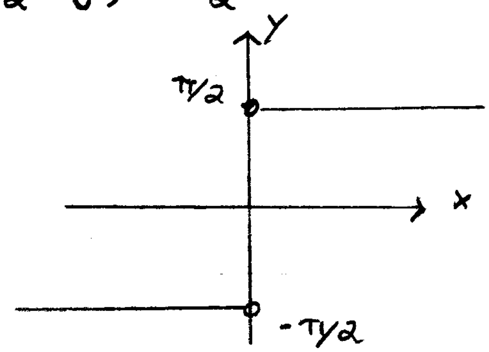
Thus $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = u + \left(\frac{\pi}{2} - u\right) = \frac{\pi}{2}$

If $x < 0$, then $-\frac{\pi}{2} < u < 0$, so $\frac{\pi}{2} > -u > 0$

$\pi > \frac{\pi}{2} - u > \frac{\pi}{2}$

$\frac{\pi}{2} - u \in QII \Rightarrow \tan^{-1}\left(\tan\left(\frac{\pi}{2} - u\right)\right) = \left(\frac{\pi}{2} - u\right) - \pi = -\frac{\pi}{2} - u$

Thus $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = u + \left(-\frac{\pi}{2} - u\right) = -\frac{\pi}{2}$



Prove $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right), \; x > 1$