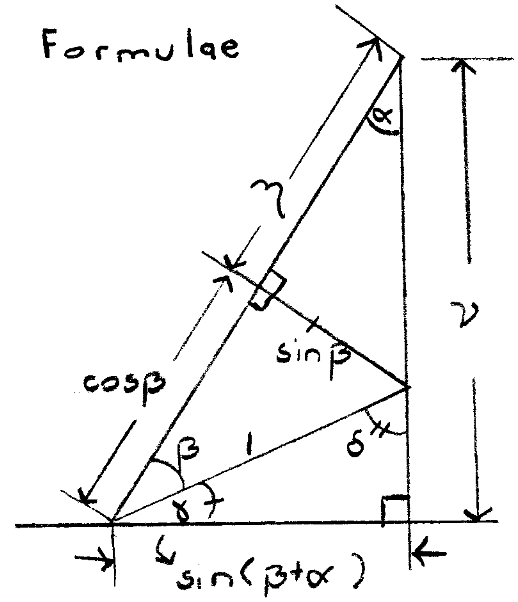
§4.2 Addition ∧ Subtraction Formulae



$\sin(\alpha+\beta)$:

$\beta+\gamma+\alpha+90 = 180 \quad (1)$

$\gamma+\delta+90 = 180 \quad (2)$

$\Rightarrow \delta = \beta+\alpha$

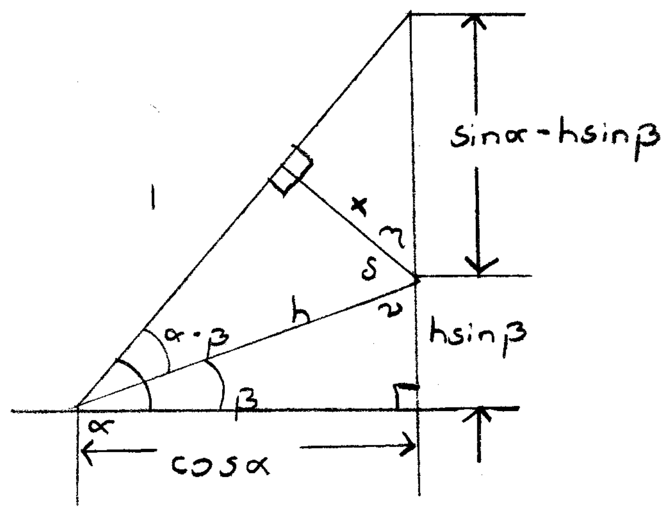
$\tan\alpha = \dfrac{\sin\beta}{\eta}$

$\eta = \dfrac{\sin\beta\cos\alpha}{\sin\alpha}$

$\sin\alpha = \dfrac{\sin(\beta+\alpha)}{\cos\beta + \dfrac{\sin\beta\cos\alpha}{\sin\alpha}}$

$\Rightarrow \sin\alpha\cos\beta + \sin\beta\cos\alpha = \sin(\beta+\alpha)$

$\sin(\alpha-\beta)$:



$\cos\beta = \dfrac{\cos\alpha}{h}$; $h = \dfrac{\cos\alpha}{\cos\beta}$

(1) $x = h\sin(\alpha-\beta)$

$\eta+\delta+\nu = 180$

$\eta+\delta+90-\beta = 180$

$\eta+(90+\beta-\alpha)+90-\beta = 180$

$\eta = \alpha$

(2) $x = (\sin\alpha - h\sin\beta)\cos\alpha$

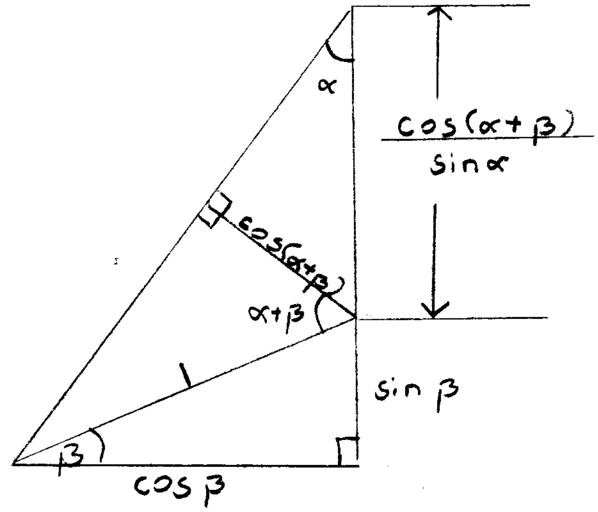
From (1) ∧ (2)

$h\sin(\alpha-\beta) = (\sin\alpha - h\sin\beta)\cos\alpha$

$\sin(\alpha-\beta) = (\sin\alpha / h - \sin\beta)\cos\alpha$

$= (\sin\alpha \dfrac{\cos\beta}{\cos\alpha} - \sin\beta)\cos\alpha$

$= \sin\alpha\cos\beta - \sin\beta\cos\alpha$

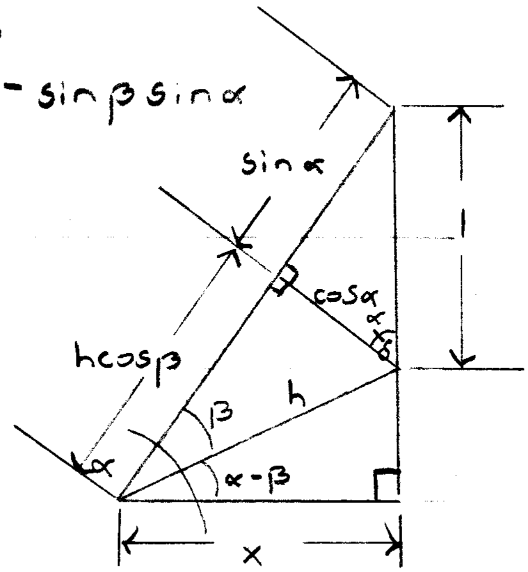


$$\frac{\sin\alpha}{\cos\alpha} = \frac{\cos\beta}{\frac{\cos(\alpha+\beta)}{\sin\alpha} + \sin\beta}$$

$$\frac{\cos(\alpha+\beta)}{\sin\alpha} + \sin\beta = \frac{\cos\alpha\cos\beta}{\sin\alpha}$$

$$\cos(\alpha+\beta) + \sin\beta\sin\alpha = \cos\alpha\cos\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\beta\sin\alpha$$



$$\delta + 90 - \beta + 90 - (\alpha - \beta) = 180$$

$$\delta = \alpha$$

$$h\sin\beta = \cos\alpha \qquad h = \frac{\cos\alpha}{\sin\beta}$$

$$x = h\cos(\alpha-\beta)$$

$$x = (h\cos\beta + \sin\alpha)\cos\alpha$$

$$h\cos(\alpha-\beta) = (h\cos\beta + \sin\alpha)\cos\alpha$$

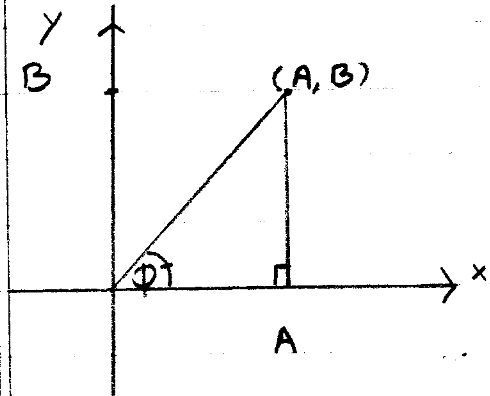
$$\cos(\alpha-\beta) = \left(\cos\beta + \frac{\sin\alpha}{h}\right)\cos\alpha$$

$$= \cos\beta\cos\alpha + \sin\alpha\sin\beta$$

$$\tan(s+t) = \frac{\sin(s+t)}{\cos(s+t)} = \frac{\sin s\cos t + \cos s\sin t}{\cos s\cos t - \sin s\sin t}$$

$$= \frac{\tan s + \tan t}{1 - \tan s\tan t}$$

Let $g(x) = \cos(x)$ Find $\frac{g(x+h) - g(x)}{h}$



$A\sin x + B\cos x =$

$\sqrt{A^2 + B^2}\left[\frac{A}{\sqrt{A^2+B^2}}\sin x + \frac{B}{\sqrt{A^2+B^2}}\cos x\right]$

$= \sqrt{A^2 + B^2}\,(\cos\phi \sin x + \sin\phi \cos x)$

$= \sqrt{A^2 + B^2}\,\sin(x + \phi)$

$\tan 195^\circ = \frac{\sin 195^\circ}{\cos 195^\circ} = \frac{\sin(135+60)}{\cos(135+60)}$

$= \frac{\sin 135 \overset{\cos}{\cancel{\sin}} 60 + \sin 60 \cos 135}{\cos 135 \cos 60 - \sin 135 \sin 60} = \frac{\frac{\sqrt{2}}{2}\cdot\frac{\overset{1}{\cancel{\sqrt{3}}}}{2} + \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right)}{-\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}}$

$= \frac{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}}{-\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}} = \frac{\sqrt{2} - \sqrt{6}}{-\sqrt{2} - \sqrt{6}}$

$1 - \tan x \tan y = \frac{\cos(x+y)}{\cos x \cos y}$

$= \frac{\tan x + \tan y}{\tan(x+y)} = \frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\sin(x+y)}{\cos(x+y)}} = \frac{\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}}{\frac{\sin x \cos y + \sin y \cos x}{\cos(x+y)}}$

$= \frac{\cos(x+y)}{\cos x \cos y}$

$3\sin\pi x + 3\sqrt{3}\cos\pi x$

$\tan\Phi = \sqrt{3}$

$\Phi = 60^\circ$

$6\sin(\pi x + 60^\circ)$

$\sim$