

§ 4,1 Trig Identities

I

Reciprocal

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

II

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

III

Even-Odd Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

IV

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u$$

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Simplify

$$\tan x \csc x \cos x = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \cos x = 1$$

$$\frac{\cos x}{\sec x + \tan x} = \frac{\cos x}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} = \frac{\cos^2 x}{1 + \sin x} = \frac{\cos^2 x (1 - \sin x)}{(1 + \sin x)(1 - \sin x)}$$
$$= 1 - \sin x$$

Proving Identities

- 1 Start w/ one side; pick one side of equation, write it down, goal is to transform it to other side
- 2 Use known identities; use algebra; combine fractions, factor, simplify
- 3 convert expressions to sines & cosines if necessary

$$\sec x + \csc x = \sin x + \cos x$$

$$\tan x + \cot x$$

$$\frac{1}{\cos x} + \frac{1}{\sin x} = \frac{\sin x}{\sin x \cos x} + \frac{\cos x}{\sin x \cos x} = \sin x + \cos x$$
$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\cos x \sin x}$$

$$\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$$

$$\left. \begin{aligned} (\sin x - \tan x)(\cos x - \cot x) &= \\ (\cos x - 1)(\sin x - 1) & \end{aligned} \right\}$$

$$\frac{\sin A}{1 - \cos A} - \cot A = \csc A$$

$$\frac{\sin A}{1 - \cos A} - \frac{\cos A}{\sin A} = \frac{\sin^2 A}{(1 - \cos A)\sin A} - \frac{\cos A(1 - \cos A)}{(1 - \cos A)\sin A} = \frac{-\cos A}{\sin A - \cos A \sin A}$$

$$\frac{\sin A}{1 - \cos A} = \frac{1}{\sin A} + \frac{\cos A}{\sin A} = \frac{1 + \cos A}{\sin A}$$

$$\sin^2 A = (1 + \cos A)(1 - \cos A) = 1 - \cos^2 A$$