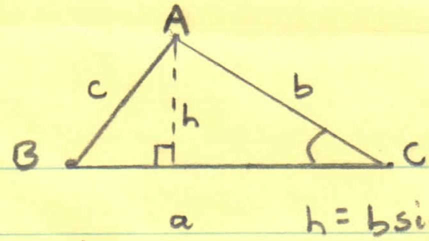
§ 3.4 Law of Sines



In $\triangle ABC$ we have

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$h = b\sin C$

Proof We have the area of $\triangle ABC$ is $\frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A$

$$\frac{2}{abc}\left[\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C\right]$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

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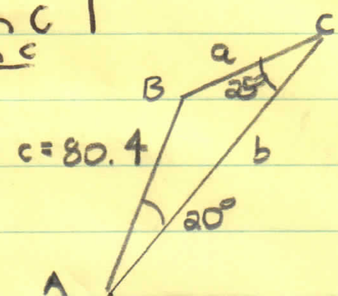
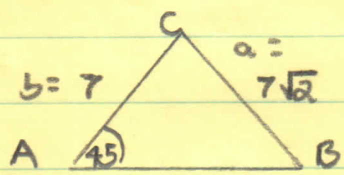


Solving a Triangle

Case I $\angle B = 180 - (20 + 25) = 135$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\quad a = \frac{c\sin A}{\sin C} = \frac{80.4\sin 20}{\sin 25} \approx 65.1$

$b = \frac{c\sin B}{\sin C} \approx 134.5$



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Case II $\sin B = \frac{b\sin A}{a} = \frac{7}{7\sqrt{2}}\sin 45^\circ = \frac{1}{2}$

$\angle B$ can be 30° or 150°. But $\angle B \neq 150^\circ$ since $\angle A = 45^\circ$

$\angle B = 30^\circ \Rightarrow \angle C = 180 - (45 + 30) = 105^\circ$

$c = \frac{b\sin C}{\sin B} = 7\cdot\sin 105^\circ / \sin 30^\circ \approx 13.5$

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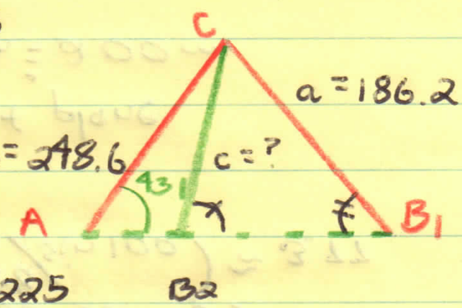


Case III Given: $\angle A = 43.1^\circ$ $a = 186.2$ $b = 248.6$

$\sin B = \frac{b\sin A}{a} = \frac{248.6\sin 43.1}{186.2} \approx 0.91225$

$\angle B_1 = \sin^{-1} 0.91225 = 65.8^\circ$. Also $\angle B_2 = 180 - 65.8 \approx 114.2^\circ$

Solve $A_1B_1C_1$: $\angle C_1 = 180 - (43.1 + 65.8) = 71.1$

$c_1 = \frac{a_1\sin C_1}{\sin A_1} \approx 257.8$

Solve $A_2B_2C_2$: $\angle C_2 = 180 - (43.1 + 114.2) = 22.7$

$c_2 = \frac{a_2\sin C_2}{\sin A_2} \approx 105.2$

Case IV $\angle A = 42^\circ$, $a = 70$ $b = 122$ (SSA)

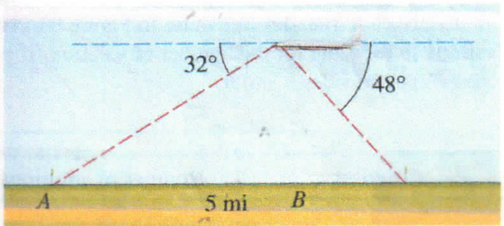
$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\sin B = \frac{b \sin A}{a} = \frac{122 \sin 42}{70} \approx 1.17$

But $\sin\theta \not> 1$! No triangle

A pilot is flying over a straight highway. He determines the angles of depression to two mileposts, 5 mi apart, to be 32° and 48°, as shown in the figure.

(a) Find the distance of the plane from point *A*.

(b) Find the elevation of the plane.



Let x be the distance from the plane to point A.

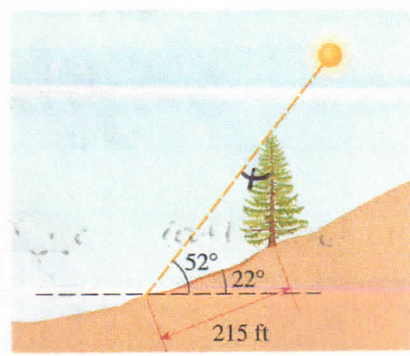
Then $\frac{x}{AB} = \frac{\sin 48}{\sin(180-32-48)}$ =>

$x = 5\left(\sin 48 / \sin 100\right) \approx 3.77$

Let h = height of plane

$\sin 32 = \frac{h}{x}$, $h \doteq 2.00$ mi.

A tree on a hillside casts a shadow 215 ft down the hill. If the angle of inclination of the hillside is 22° to the horizontal and the angle of elevation of the sun is 52°, find the height of the tree.



Assume that the tree is growing $\perp$ to flat ground $\wedge$ not the hillside. Then the angle subtended by the top of the tree and the sun's ray is

$\angle A = 180 - 90 - 52 = 38$

$\therefore$ height of tree : $h = \frac{215 \sin 30^\circ}{\sin 38^\circ}$

$\approx 175'$

334 {all evens 2-32}

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