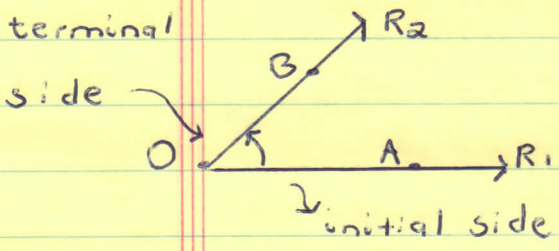
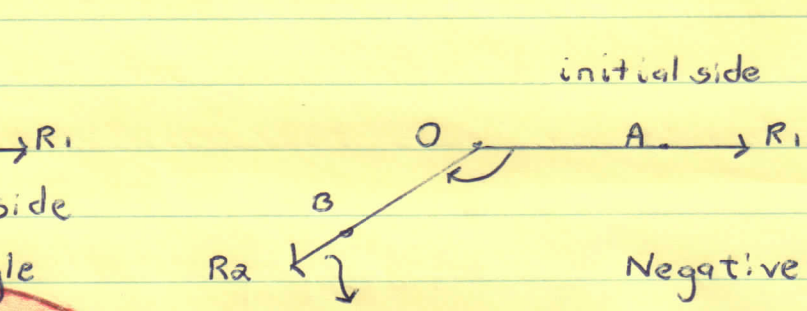
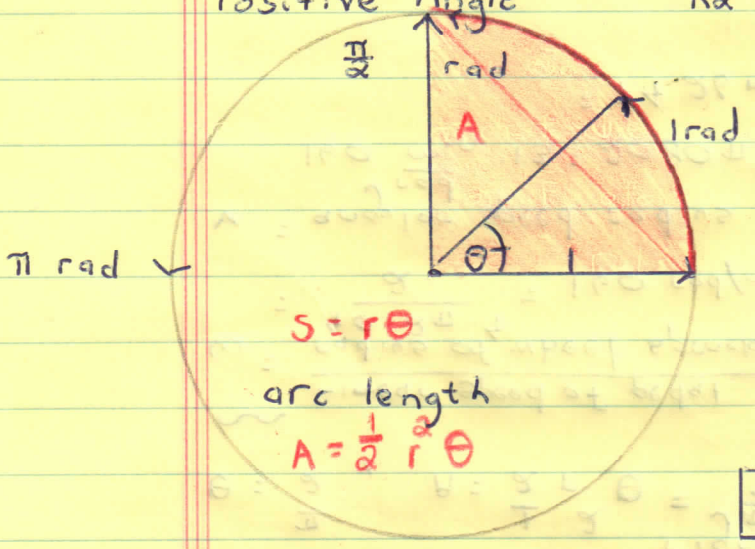
Chapter 3: Trig Functions of Angles



Positive Angle



Negative Angle



$s = r\theta$

arc length

$A = \frac{1}{2} r^2 \theta$

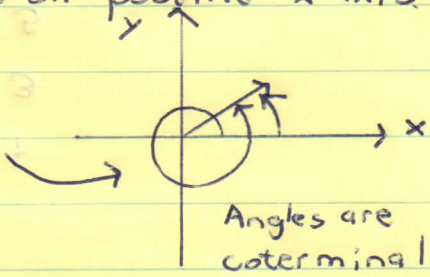
circle of radius 1 drawn w/ the vertex of an angle at its center, measure of angle in radians (rad) := length of arc that subtends angle

$180^\circ = \pi$ rad

An angle is in standard position if it is drawn in xy-plane w/ vertex at origin and initial side on positive x-axis

Two angles in standard position are coterminal if their sides coincide

* See examples pp 179-180



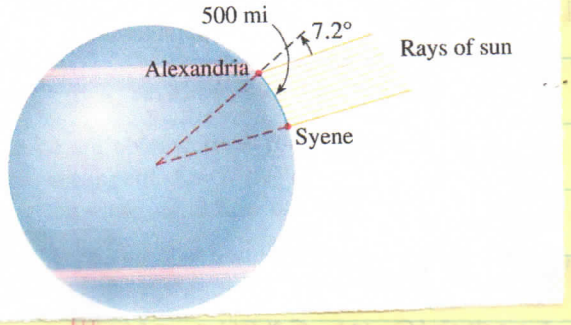
If a point moves along a circle of radius r and the ray from the center of the circle to the point traverses θ rads in time t. Let $s = r\theta$ be the distance the point travels in time t. Angular speed $\omega = \frac{\theta}{t}$ Linear speed $v = \frac{s}{t}$

$v = r\omega$

#36 Since $-\frac{7\pi}{3} - \frac{5\pi}{3} = 4\pi$ and $2\pi \mid 4\pi$, the angles $-\frac{7\pi}{3}$ & $\frac{5\pi}{3}$ are coterminal

#38 10 and $10 - 2\pi$ are coterminal.

The Greek mathematician Eratosthenes (ca. 276–195 B.C.) measured the circumference of the earth from the following observations. He noticed that on a certain day the sun shone directly down a deep well in Syene (modern Aswan). At the same time in Alexandria, 500 miles north (on the same meridian), the rays of the sun shone at an angle of 7.2° to the zenith. Use this information and the figure to find the radius and circumference of the earth. (The data used in this problem are more accurate than those available to Eratosthenes.)



Since the sun is so far away, assume that the rays of the sun are parallel when striking the earth, thus the angle formed at the center of the earth is also $\theta = 7.2^\circ$

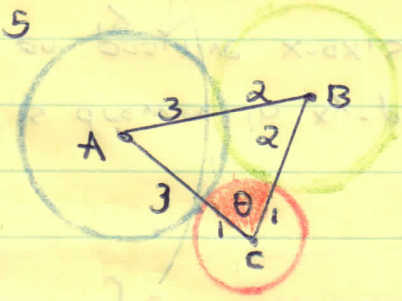
$$r = \frac{s}{\theta} = \frac{500}{7.2 \cdot \pi/180} \approx 3980 \text{ mi}$$

$$c = 2\pi r = \frac{2\pi \cdot 180 \cdot 500}{7.2\pi} = 25{,}000$$

$AC = 3 + 1 = 4$

$BC = 1 + 2 = 3$

$AB = 2 + 3 = 5$



$AB^2 = AC^2 + BC^2 \Rightarrow$ the triangle is a right triangle!

$\theta = \frac{\pi}{2}$, $A = \frac{1}{2} r^2 \theta = \frac{\pi}{4}$ ft^2

$\omega = \frac{\text{Linear speed of pedal}}{\text{radius of wheel sprocket}}$

$= \frac{40 \cdot 2\pi \cdot 4}{2} = 160$ rad/min

$v = \text{angular speed} \cdot \text{radius}$

$160 \frac{\text{rad}}{\text{min}} \cdot 13 = 2080\pi$ in/min

$= 74.26$ mph

The sprockets and chain of a bicycle are shown in the figure. The pedal sprocket has a radius of 4 in, the wheel sprocket a radius of 2 in, and the wheel a radius of 13 in. The cyclist pedals at 40 rpm.

(a) Find the angular speed of the wheel sprocket.

(b) Find the speed of the bicycle. (Assume that the wheel turns at the same rate as the wheel sprocket.)