Math 201, Final Exam, Summer 2013

You are required to show all work. A response based on output from a calculator only will receive no credit.

Email your solutions to rfrith@uaa.alaska.edu no later than 5:00 PM, Friday, August 2, 2013, or, deliver your paper-based solutions to my mailbox at the Eagle River Campus.

Email attachments must be in the form of a single PDF document. Any other format will be rejected and you will not receive credit for the final.

Name____

Q1. Evaluate the following integrals:

a)

$$2\int x^3 \cos(x^2)\, dx$$

b)

$$\int e^{2x} \cos(x)\, dx$$

c)

$$\int \frac{x}{(x+1)(x^2+4)}$$

d)

$$\int \sin^3(x) \cos^2(x)\, dx$$

e)

$$\int_0^\infty xe^{-2x}\, dx$$

Q2. Determine which of the following improper integrals exist.

a) $\int_0^{\infty} \frac{x^2-1}{(x^2+x+1)^{3/2}}\,dx$

b) $\int_1^{\infty} \frac{1}{x^2+1}\,dx$

c) $\int_2^{5} \frac{dx}{\sqrt{x-2}}$.

Q3. State if the following sequences are convergent or divergent. If the sequence converges, then determine the limit.

a) $a_n = \frac{2n^2+3n-1}{5n^2+2}$.

b) $a_n = 2n\sin(1/n)$.

c) $\sqrt{n+2}-\sqrt{n}$.

Q4. State if the following sequences are convergent or divergent. Justify your answers.

a) $\sum_{n=1}^{\infty} \frac{n}{n^2+2n+1}$

b) $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{2}{n+1}$

c) $\sum_{n=0}^{\infty} n^2 e^{-n}$

d) $\sum_{n=1}^{\infty} n!5^{-n}$.

Q5. Evaluate the infinite sum.

Evaluate the infinite sums $\sum_{n=2}^{\infty} 4 \cdot 2^{-n}$

Q6. Determine the radius of convergence of the following power series.

a) $\sum_{n=1}^{\infty} \frac{(x+1)^n}{12^n}$. $R =$

b) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{(2n)!}$. $R =$

Q7. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{4^n}{n^2}(x-1)^n$. Remember to consider the endpoints!

Q8.

Find the Taylor polynomial $T_4(x)$ for $\cos(x)$ at $c = \pi/4$.

Q9. Recall that the derivative of $\tan^{-1}(x)$ is $\frac{1}{1+x^2}$. Find the power series at $c = 0$ and the radius of convergence for

$f(x) = x \tan^{-1}(x) =$

Q10. Let $c(t) = (t^2 + 1, t^2 - 2t)$.

a) Find the x and y coordinates at time $t = 2$. $x =$ __________, $y =$ __________

b) Evaluate $\frac{dy}{dx}$ at the point $t = 2$. $\frac{dy}{dx} =$ __________.

c) The equation of the tangent line through the point $c(2)$ is ____ $y =$ ____ $x +$ ____

Q11. The curve $r = \frac{10}{2\cos(\theta) - \sin(\theta)}$ represents a line. The equation of the line in form of x and y is: ____ $y =$ ____ $x +$ ____

Q12. A circle C has center at the origin and radius 2. The circle K has radius 2 and center at the point $(0, 2)$.

a) Write the equation of both circles in polar coordinates.
C has the equation $r =$ ______________ and K has the equation $r =$ ______________

b) Find the x, y coordinates of the points where the two circles intersects. $x =$ __________, $y =$ __________

c) Set up the integral representing the area **outside** the circle C and **inside** the circle K.

Q13. Use the method of Cylindrical Shells to find the volume of the solid obtained by rotating the region enclosed by $y = x^3$, $y = \sqrt[3]{x}$ for $x \geq 0$ about the <u>y - axis</u>.