HW #1

1 graph $y = f(x) = \dfrac{x^2 - 4}{x^2 - 1}$

a) symmetry: since x is raised to even powers only, $f(-x) = f(x) \Rightarrow$ symmetry about y-axis. No symmetry about origin.

b) vertical asymptotes

$f(x) = \dfrac{x^2 - 4}{x^2 - 1} = \dfrac{(x+2)(x-2)}{(x+1)(x-1)}$ $\quad (x+1)(x-1) = 0$

$x = -1$ or $x = 1$

c) intercepts

$f(0) = 4$, $(0, 4)$ $\quad y\text{-} = (0, 4)$

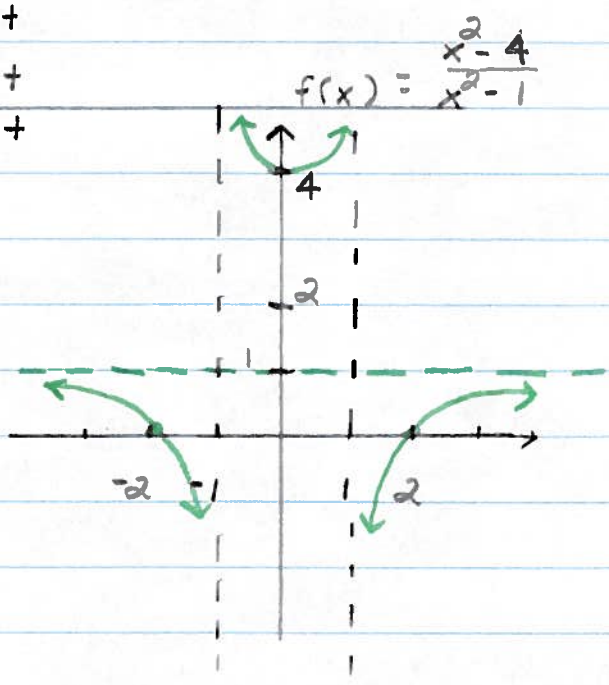
$x\text{-} = (-2, 0) \wedge (2, 0)$

d) horizontal asymptotes:

$\deg(P(x)) = \deg(Q(x)) \Rightarrow y = 1$ is an H.A.

e) above/below x-axis

$x+2$	−	+	+	+	+
$x-2$	−	−	−	−	+
$x+1$	−	−	+	+	+
$x-1$	−	−	−	+	+
$f(x)$	+	−	+	−	+
		-2	-1	1	2



2 a) $\ln(\ln(x)) = 1$

$\ln(x) = e$

$x = e^e$

b) $e^{2x-3} - 7 = 0$

$e^{2x-3} = 7$

$2x - 3 = \ln 7$

$2x = 3 + \ln 7$

$x = \frac{3+\ln 7}{2}$

c) $|x-1| - |x-3| \geq 5$

$$|x-1| = \begin{cases} -x+1 & \text{if } x-1 < 0 \\ x-1 & \text{if } x-1 \geq 0 \end{cases}$$

$$|x-3| = \begin{cases} -x+3 & \text{if } x-3 < 0 \\ x-3 & \text{if } x-3 > 0 \end{cases}$$

case 1: $x < 1$

$-x+1 - (-x+3) \geq 5$

$4 \geq 5$ ↯

case 2: $1 \leq x < 3$

$x-1 - (-x+3) \geq 5 \Rightarrow 2x - 4 \geq 5 \Rightarrow x \geq \frac{9}{2}$

but $1 < x < 3$, so $x \geq \frac{9}{2}$ is not a solution ↯

case 3: $x \geq 3$

$x-1 - (x-3) \geq 5 \Rightarrow 2 \geq 5$ ↯

∴ no solution

d) $e^{ax} = ce^{bx} \Rightarrow e^{x(a-b)} = c \Rightarrow x(a-b) = \ln c$

$= \frac{\ln c}{a-b}$

e) $\ln(x) + \ln(x-1) = 1 \Rightarrow e^{[\ln(x) + \ln(x-1)]} = e$

$e^{\ln x} e^{\ln(x-1)} = e$

$x(x-1) = e$

$x^2 - x - e = 0 \Rightarrow x_{1,2} = \frac{1 \pm \sqrt{1+4e}}{2}$

3 $y = (x - \frac{1}{3}y)^{-4} \Rightarrow y = \frac{1}{(x - \frac{1}{3}y)^4}$ use binomial theorem

$$y = \frac{1}{x^4 + 4x^3(-\frac{y}{3}) + 6x^2(\frac{y}{3})^2 + 4x(\frac{y}{3})^3 + (\frac{y}{3})^4}$$

4 $f(x) = x + 4, \quad h(x) = 4x - 1$

$g \circ f = h$ $\qquad g(f(x)) = 4x - 1$

$g(x+4) = 4x - 1$

$g((x+4)-4) = 4(x-4) - 1$

$g(x) = 4x - 17$

5 $H(x) = \sqrt[8]{2+|x|} = f \circ g \circ h$

$f(x) = \sqrt[8]{x} \qquad g(x) = 2 + x \qquad h(x) = |x|$

6 $y = ax^2 + bx + c \qquad \{(0,-1), (-1,-4), (1,-2)\}$

$-1 = c$ ← plug in

$-4 = a - b + c \Rightarrow -4 = a - b - 1 \Rightarrow -3 = a - b \Rightarrow a = -2$

$-2 = a + b + c \Rightarrow -2 = a + b - 1 \Rightarrow -1 = a + b \qquad b = 1$

$y = -2x^2 + x - 1$

7 $f(x) = \dfrac{x+3}{x+1} \qquad \dfrac{f(x) - f(1)}{x-1} = \dfrac{\frac{x+3}{x+1} - 2}{x-1}$

$= \dfrac{x+3-2x-2}{(x+1)(x-1)} = \dfrac{-x+1}{(x+1)(x-1)} = \dfrac{-1}{x+1}$

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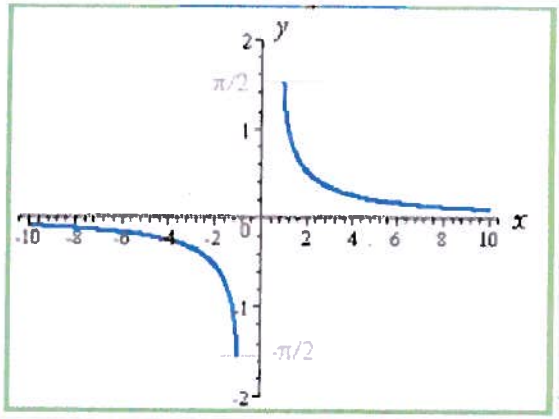

$y = \operatorname{arccsc}(x)$