A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where $t$ is measured in seconds and $s$ in feet.

$$f(t) = t^3 - 9t^2 + 15t$$

(a) Find the velocity at time $t$.

$$v(t) = \frac{ds}{dt} = 3t^2 - 18t + 15$$

(b) What is the velocity after 4 s?

$$v(4) = -9 \text{ ft/s}$$

(c) When is the particle at rest?

$$t = 1 \text{ s (smaller value)}$$

$$t = 5 \text{ s (larger value)}$$

(d) When is the particle moving in the positive direction? (Enter your answer in interval notation.)

$$t \in [0, 1) \cup (5, \infty)$$

(e) Find the total distance traveled during the first 8 s.

$$120 \text{ ft}$$

(f) Find the acceleration at time $t$.
\[ a(t) = 6t - 18 \]

Find the acceleration after 4 s.
\[ a(4) = 6 \text{ ft/s}^2 \]

(g) Graph the position, velocity, and acceleration functions for \( 0 \leq t \leq 8 \).
(h) When is the particle speeding up? (Enter your answer in interval notation.)

\[ (1, 3) \cup (5, \infty) \]

When is it slowing down? (Enter your answer in interval notation.)

\[ [0, 1) \cup (3, 5) \]
When is the particle in figure (a) speeding up? (Enter your answer using interval notation.)

\((0, 1) \cup (2, 3)\)

When is the particle in figure (a) slowing down? (Enter your answer using interval notation.)

\((1, 2)\)

When is the particle in figure (b) speeding up? (Enter your answer using interval notation.)

\((1, 2) \cup (3, 4)\)

When is the particle in figure (b) slowing down? (Enter your answer using interval notation.)

\((0, 1) \cup (2, 3)\)
If a ball is thrown vertically upward with a velocity of 160 ft/s, then its height after $t$ seconds is $s = 160t - 16t^2$.

(a) What is the maximum height reached by the ball?

\[400\text{ ft}\]

(b) What is the velocity of the ball when it is 384 ft above the ground on its way up? (Consider up to be the positive direction.)

\[32\text{ ft/s}\]

What is the velocity of the ball when it is 384 ft above the ground on its way down?

\[-32\text{ ft/s}\]
4. 0/4 points

(a) Find the average rate of change of the area of a circle with respect to its radius $r$ as $r$ changes from 3 to each of the following.

(i) 3 to 4

$$7\pi$$

(ii) 3 to 3.5

$$6.5\pi$$

(iii) 3 to 3.1

$$6.1\pi$$

(b) Find the instantaneous rate of change when $r = 3$.

$$A'(3) = 6\pi$$
5. 0/3 points

A spherical balloon is being inflated. Find the rate of increase of the surface area \( S = 4\pi r^2 \) with respect to the radius \( r \) when \( r \) is each of the following.

(a) 3 ft

\[
\frac{24\pi}{ft} \text{ ft}^2/ft
\]

(b) 4 ft

\[
\frac{32\pi}{ft} \text{ ft}^2/ft
\]

(c) 5 ft

\[
\frac{40\pi}{ft} \text{ ft}^2/ft
\]

Solution or Explanation

Click to View Solution

Need Help?  
Read It  
Watch It

6. 0/4 points

Newton's Law of Gravitation says that the magnitude \( F \) of the force exerted by a body of mass \( m \) on a body of mass \( M \) is

\[
F = \frac{GmM}{r^2}
\]

where \( G \) is the gravitational constant and \( r \) is the distance between the bodies.

(a) Find \( dF/dr \).
\[ \frac{dF}{dr} = \frac{2GmM}{r^3} \]

What is the meaning of \( \frac{dF}{dr} \)?

- \( \frac{dF}{dr} \) represents the amount of force per distance.
- \( \frac{dF}{dr} \) represents the rate of change of the distance between the bodies with respect to the force.
- \( \frac{dF}{dr} \) represents the rate of change of the mass with respect to the force.
- \( \frac{dF}{dr} \) represents the rate of change of the mass with respect to the distance between the bodies.
- \( \frac{dF}{dr} \) represents the rate of change of the force with respect to the distance between the bodies.

What does the minus sign indicate?

- The minus sign indicates that as the distance between the bodies decreases, the magnitude of the force remains constant.
- The minus sign indicates that the force between the bodies is decreasing.
- The minus sign indicates that as the distance between the bodies increases, the magnitude of the force increases.
- The minus sign indicates that the bodies are being forced in the negative direction.
- The minus sign indicates that as the distance between the bodies increases, the magnitude of the force decreases.

(b) Suppose it is known that the earth attracts an object with a force that decreases at the rate of 2 N/km when \( r = 30,000 \) km. How fast does this force change when \( r = 15,000 \) km?

-16 N/km
Some of the highest tides in the world occur in the Bay of Fundy on the Atlantic Coast of Canada. At Hopewell Cape the water depth at low tide is about 2.0 m and at high tide it is about 12.0 m. The natural period of oscillation is a little more than 12 hours and on June 30, 2009, high tide occurred at 6:45 AM. This helps explain the following model for the water depth $D$ (in meters) as a function of the time $t$ (in hours after midnight) on that day:

$$D(t) = 7 + 5 \cos[0.503(t - 6.75)].$$

How fast was the tide rising (or falling) at the following times? (Round your answers to two decimal places.)

(a) 4:00 AM

$$D'(4) = -5 \sin[0.503(4 - 6.75)](0.503) = -2.515 \sin[0.503(-2.75)] \approx -2.12 \text{ m/h (falling)}.$$  

(b) 6:00 AM

$$D'(6) = -5 \sin[0.503(6 - 6.75)](0.503) = 0.93 \text{ m/h (rising)}.$$  

(c) 10:00 AM

$$D'(10) = -5 \sin[0.503(10 - 6.75)](0.503) = -2.51 \text{ m/h (falling)}.$$  

(d) 11:00 AM

$$D'(11) = -5 \sin[0.503(11 - 6.75)](0.503) = 0.93 \text{ m/h (rising)}.$$
(b) At 6:00 AM, \( t = 6 \), and \( D'(6) = -2.515 \sin[0.503(-0.75)] \approx 0.93 \text{ m/h (rising)} \).

(c) At 10:00 AM, \( t = 10 \), and \( D'(10) = -2.515 \sin[0.503(3.25)] \approx -2.51 \text{ m/h (falling)} \).

(d) At 11:00 AM, \( t = 11 \), and \( D'(11) = -2.515 \sin[0.503(4.25)] \approx -2.12 \text{ m/h (falling)} \).
The cost function for production of a commodity is

\[ C(x) = 335 + 25x - 0.05x^2 + 0.0005x^3. \]

(a) Find \( C'(100) \).

\( 30 \)

Interpret \( C'(100) \).

- This is the rate at which costs are increasing with respect to the production level when \( x = 100 \).
- This is the amount of time, in minutes, it takes to produce 100 items.
- This is the cost of making 100 items.
- This is the number of items that must be produced before the costs reach 100.
- This is the rate at which the production level is decreasing with respect to the cost when \( x = 100 \).

(b) Find the actual cost of producing the 101st item. (Round your answer to the nearest cent.)

\( 30.10 \)
9. 0/1 points  
SCalc7 2.8.004.MI. [1874755]  
The length of a rectangle is increasing at a rate of 3 cm/s and its width is increasing at a rate of 9 cm/s. When the length is 9 cm and the width is 7 cm, how fast is the area of the rectangle increasing?  

\[
\text{Area} = 102 \text{ cm}^2/\text{s}
\]

Solution or Explanation  
Click to View Solution

Need Help?  
Read It  Watch It  Master It

10. 0/1 points  
SCalc7 2.8.005.MI. [1874494]  
A cylindrical tank with radius 7 m is being filled with water at a rate of 2 m\(^3\)/min. How fast is the height of the water increasing?  

\[
\frac{2}{49\pi} \text{ m/min}
\]

Solution or Explanation  
Click to View Solution

Need Help?  
Read It  Watch It  Master It
11. 0/1 points

A plane flying horizontally at an altitude of 2 mi and a speed of 450 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 3 mi away from the station. (Round your answer to the nearest whole number.)

Solution or Explanation

\[ \frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{x}{y} (450). \quad \text{Since } y^2 = x^2 + 4, \quad \text{when } y = 3, \quad x = \sqrt{5}, \quad \text{so } \frac{dy}{dt} = \frac{5}{3}(450) = 150\sqrt{5} \approx 335 \text{ mi/h.} \]

Need Help?  
[Read It]  [Watch It]

12. 0/1 points

At noon, ship A is 120 km west of ship B. Ship A is sailing east at 20 km/h and ship B is sailing north at 15 km/h. How fast is the distance between the ships changing at 4:00 PM?

Solution or Explanation

At 4:00 PM, \( x = 4(20) = 80 \) and \( y = 4(15) = 60 \) \( \Rightarrow \) \( z = \sqrt{(120 - 80)^2 + 60^2} = 20\sqrt{13} \).

So \( \frac{dz}{dt} = \frac{1}{z} \left[ (x - 120) \frac{dx}{dt} + y \frac{dy}{dt} \right] = \frac{(-40)(20) + 60(15)}{20\sqrt{13}} = \frac{5}{\sqrt{13}} \approx 1.3868 \text{ km/h.} \)

Need Help?  
[Read It]
Water is leaking out of an inverted conical tank at a rate of 8,500 cm³/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank. (Round your answer to the nearest integer.)

\[ \text{Rate at which water is pumped into the tank} \]

\[ 287,753 \text{ cm}^3/\text{min} \]
14. 0/1 points

If two resistors with resistances $R_1$ and $R_2$ are connected in parallel, as in the figure below, then the total resistance $R$, measured in ohms ($\Omega$), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If $R_1$ and $R_2$ are increasing at rates of 0.3 $\Omega$/s and 0.2 $\Omega$/s, respectively, how fast is $R$ changing when $R_1 = 70$ $\Omega$ and $R_2 = 90$ $\Omega$? (Round your answer to three decimal places.)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Solution or Explanation

Click to View Solution

Need Help? Read It Watch It
15. 0/1 points  

A lighthouse is located on a small island 4 km away from the nearest point \( P \) on a straight shoreline and its light makes eight revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from \( P \)? (Round your answer to one decimal place.)

\[
\text{km/min} \quad 213.6
\]

Solution or Explanation  
Click to View Solution  
Need Help? Read It

16. 0/1 points  

Find the linearization \( L(x) \) of the function at \( a \).

\[
f(x) = x^{2/3}, \quad a = 64
\]

\[
L(x) = \frac{16}{3} + \frac{1}{6}x
\]

Solution or Explanation  
Click to View Solution  
Need Help? Read It

17. 0/4 points  

Find the linear approximation of the function \( g(x) = \sqrt[3]{1 + x} \) at \( a = 0 \).

\[
g(x) \approx \frac{1}{3}x + 1
\]

Solution or Explanation  
Click to View Solution  
Need Help? Read It
Use it to approximate the numbers $\sqrt[3]{0.95}$ and $\sqrt[3]{1.1}$. (Round your answers to three decimal places.)

$\sqrt[3]{0.95} \approx 0.983$

$\sqrt[3]{1.1} \approx 1.033$

Illustrate by graphing $g$ and the tangent line.
Solution or Explanation

\[ g(x) = \sqrt[3]{1 + x} = (1 + x)^{1/3} \implies g'(x) = \frac{1}{3}(1 + x)^{-2/3}, \] so

\[ g(0) = 1 \quad \text{and} \quad g'(0) = \frac{1}{3}. \]

Therefore, \( \sqrt[3]{1 + x} = g(x) \approx g(0) + g'(0)(x - 0) = 1 + \frac{1}{3}x. \)

The linear approximation \( 1 + \frac{1}{3}x \) to \( f \) at \( x = 0 \) provides a good approximation to \( f \) for \( x \) near 0.

So \( \sqrt[3]{0.95} = \sqrt[3]{1 + (-0.05)} = 1 + \frac{1}{3}(-0.05) = 0.983 \)

and \( \sqrt[3]{1.1} = \sqrt[3]{1 + 0.1} = 1 + \frac{1}{3}(0.1) = 1.033. \)

Graphing \( f \) and the tangent line we have:
18. 0/1 points

Verify the given linear approximation at \( a = 0 \). Then determine the values of \( x \) for which the linear approximation is accurate to within 0.1. (Enter your answer using interval notation. Round your answers to two decimal places.)

\[
9 \tan x \approx 9x
\]

\[ x \in (-0.32, 0.32) \]
19. 0/2 points

Find the differential of each function.

(a) \[ y = \frac{s}{1 + 5s} \]
\[ dy = \frac{1}{(1 + 5s)^2} ds \]

(b) \[ y = u \cos u \]
\[ dy = (\cos u - u \sin u) \, du \]

Solution or Explanation
Click to View Solution

Need Help? Read It

20. 0/3 points

Compute \( \Delta y \) and \( dy \) for the given values of \( x \) and \( dx = \Delta x \). (Round your answers to three decimal places.)

\( y = 2x - x^2, \ x = 2, \ \Delta x = -0.6 \)
\[ \Delta y = \boxed{0.840} \]
\[ dy = \boxed{1.200} \]

Sketch a diagram showing the line segments with lengths \( dx, dy, \) and \( \Delta y \).
Solution or Explanation

\[ y = f(x) = 2x - x^2, \quad x = 2, \quad \Delta x = -0.6 \implies \]

\[ \Delta y = f(1.4) - f(2) = 0.840 - 0 = 0.840 \]

\[ dy = (2 - 2x) \, dx = (2 - 4)(-0.6) = 1.200 \]
The edge of a cube was found to be 15 cm with a possible error in measurement of 0.2 cm. Use differentials to estimate the maximum possible error, relative error, and percentage error in computing the volume of the cube and the surface area of the cube. (Round your answers to four decimal places.)

(a) the volume of the cube

<table>
<thead>
<tr>
<th>maximum possible error</th>
<th>135 cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>relative error</td>
<td>0.0400</td>
</tr>
<tr>
<td>percentage error</td>
<td>4.0000 %</td>
</tr>
</tbody>
</table>

(b) the surface area of the cube

<table>
<thead>
<tr>
<th>maximum possible error</th>
<th>36 cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>relative error</td>
<td>0.0267</td>
</tr>
<tr>
<td>percentage error</td>
<td>2.6667 %</td>
</tr>
</tbody>
</table>

Solution or Explanation

Click to View Solution

Need Help? Read It Watch It
22. 0/4 points

The circumference of a sphere was measured to be 80 cm with a possible error of 0.5 cm.

(a) Use differentials to estimate the maximum error in the calculated surface area. (Round your answer to the nearest integer.)

\[ 25 \text{ cm}^2 \]

What is the relative error? (Round your answer to three decimal places.)

\[ 0.013 \]

(b) Use differentials to estimate the maximum error in the calculated volume. (Round your answer to the nearest integer.)

\[ 162 \text{ cm}^3 \]

What is the relative error? (Round your answer to three decimal places.)

\[ 0.019 \]
Find the differential of each function.

(a) \( y = \frac{u + 5}{u - 5} \)

\[
dy = \frac{10}{(u - 5)^2} du
\]

(b) \( y = (1 + r^4)^{-2} \)

\[
dy = \frac{-8r^3}{(1 + r^4)^3} dr
\]
24. 0/2 points

(a) Find the differential $dy$.

$$y = \tan x$$

$$dy = \sec^2(x) \, dx$$

(b) Evaluate $dy$ for the given values of $x$ and $dx$.

$$x = \pi/4 \quad \text{and} \quad dx = -0.05.$$ 

$$dy = -0.1$$

Solution or Explanation

Click to View Solution

Need Help?  Read It  Watch It

Assignment Details

Name (AID): E3_2 (5510670)
Submissions Allowed: 2
Category: Exam
Code:
Locked: Yes
Author: Frith, Russell ( afrgf@uaa.alaska.edu)
Last Saved: Mar 8, 2014 05:41 PM AKST
Permission: Protected
Randomization: Person
Which graded: Last

Feedback Settings

Before due date
Question Score
Assignment Score
Question Part Score
Mark
Response
Save Work
After due date
Question Score
Assignment Score
Publish Essay Scores
Key
Question Part Score
Solution